The modern parallax:
the redshift-isotropy floor, the empirical forcing of cosmic time and uniform expansion, and the measured resolution of the relativity of simultaneity

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Abstract

The standard cosmological model installs a global cosmic time, a preferred rest frame, and a spatially uniform expansion as kinematical assumptions, prior to and independent of the dynamical fit to data. We show that these assumptions are not conventions the data merely tolerate but structures the data force. The cosmological redshift is the path integral of the expansion rate, $\ln(1+z) = \int H\, dt$, so the observed anisotropy of the cosmic microwave background (CMB) separates exactly into a source term, fixed at last scattering, and a cumulative term, set by the integrated expansion along each line of sight. Granted even perfect homogeneity at last scattering, the observed isotropy of the monopole is then a direct measurement of uniform expansion—so the common supposition that the isotropy is supplied by homogeneity at decoupling is a category error. We make the alternative quantitative: were the expansion rate to track the inhomogeneous matter distribution region by region, with no single global scale factor, the accumulated redshift would scatter by $\sim 10^{-3}$ across the sky, against an observed $\lesssim 3 \times 10^{-6}$—an exclusion by three orders of magnitude. The effect is distinct from the Sachs–Wolfe anisotropies and not a re-derivation of them: when lumpiness clusters upon a single background, a photon's gravitational redshift descending into a potential well is undone climbing out, so the contributions *telescope* along the path and leave no $\sqrt{N}$ accumulation, whereas a genuine differential expansion accumulates because there is nothing to cancel against—so the observed isotropy selects the telescoping picture over the accumulating one. Every choice in the estimate biases it downward, so the number is a floor. A differential programme can evade it only by requiring the local rates to average to a common value in every direction, which is itself a global uniform expansion in a cosmic time—the very structure such programmes set out to dispense with. What is excluded is an inhomogeneous expansion *rate*; the matter lumpiness is untouched, the density being the bend of the spatial cut, so uniform expansion and lumpy matter are consistent. The remaining, finely tuned, observer-centred escape is closed by the Copernican principle together with the independently measured isotropy of the expansion history. What the exclusion selects is a global, time-ordered expansion—the cosmic foliation made dynamical—which is logically prior to the notions of space, isotropy, and homogeneity the model otherwise assumes. The same datum thus establishes, from the bottom up, the cosmic time, the uniformity of its advance, and—within the observable region, under the Copernican principle—the maximal symmetry of its slices. Read ontologically, this measured foliation is the *lapse* of the 3+1 split—an objective rate of advance, the cosmic present—while the relativity of synchrony is the *shift*; the century-old reading of synchrony's relativity as the absence of any objective present is thereby a modal fallacy, which the measured isotropy falsifies outright. We prove that fixing the physical foliation and reading it so is both necessary and sufficient for a coherent formal description of an existing, evolving world—an augmentation of general relativity that changes none of its equations, its necessary half here *measured. We also set down the history the measurement closes.* The structure this datum forces was posited by Einstein in February 1917—explicitly "against the spirit of relativity", and on the empirical ground that stellar proper motions are small compared with c—countered a month later by de Sitter, who placed the choice among candidate universes outside physical argument altogether and pressed as his sharpest objection that in Einstein's solution "time has a separate position"; and defended in 1920 by Eddington on geometric grounds, in the declared absence

of any experimental knowledge on cosmical scales. *Einstein then went nearly silent on cosmology for the rest of his life*, and never addressed how the cosmic time his own assumption distinguished stands to the relativity of simultaneity. *The assumption was correct*, and the objection was correct as a description of the structure though not as a reason to reject it. This is the empirical counterpart of the conceptual results of the companion papers [3, 5]: as stellar parallax converted the Earth's motion from an interpretive option into a measured fact, the isotropy of the cosmological redshift converts the cosmic foliation from a modelling convenience into a measured feature of the world.

1 Introduction

The standard model of cosmology is assembled in a definite order [18, 19, 20, 21]. One first posits that the average rest frame of matter—made manifest in the redshift–distance relation [16]—defines a coherent cosmic time, the one direction in which the four-dimensional world is, in Eddington's phrase, not curved [17]; then that space is the family of hypersurfaces orthogonal to it; then that this space is isotropic; then that it is homogeneous; and only last that its geometry evolves by a scale factor $a(t)$ whose form the field equations fix through the energy content [23, 24]. The first four are kinematical posits; the dynamical fit comes fifth and last. The standard presentation treats the kinematical posits as idealizations vindicated after the fact by the success of the fit—a global cosmic time, a preferred rest frame, and a uniform expansion accepted because the model built on them works.

The claim of this paper is that observation forces those kinematical posits directly, and not through the success of the dynamical fit. The forcing is a falsification, not a confirmation: the one coherent alternative to the posits is given its observable consequence and rejected by the data. Where the standard argument runs only the affirmative half of the scientific loop—the model is consistent with uniform expansion—we run the half that decides, deriving what the sky would look like if the posits failed and finding that it does not look like that, by three orders of magnitude. The decisive observable is the isotropy of the CMB monopole redshift, read correctly: not as a property inherited from the emitting surface, but as the accumulated record of the expansion the radiation traversed. That this is a distinct effect from the Sachs–Wolfe anisotropies, and not a re-derivation of them, is the crux and is established in §3: lumpiness clustering upon a single background carries a metric potential whose contributions *telescope* along a photon's path—the redshift descending into a well undone climbing out—leaving no accumulation, whereas a differential expansion accumulates because there is nothing to cancel against. The discriminator is that the expansion *rate* is forced by the observation while the matter content is not, and the two must not be run together.

This is the modern counterpart of a familiar episode. Before Bessel, whether the Earth moved was, on a strict reading, an interpretive choice among empirically equivalent pictures; stellar parallax converted it into a measured fact [36], and the heliocentric frame from an option into the one the measurement singled out [11]. A purely relational reading of relativity holds, by the same token, that a preferred cosmic rest frame could never be observable even if it existed. The CMB is the measurement that overturns that expectation: the universe furnished, through the physics that produced the background and the geometry of its uniform expansion, a direct measurement of our motion with respect to the cosmic rest frame, and with it of the cosmic time that frame carries [9, 10]. What follows makes that "modern parallax" quantitative.

A real, global cosmic time is a structure general relativity's formalism admits but does not supply, and a companion paper takes it as a commitment on other grounds [3]. This paper shows that the commitment is not merely admissible but empirically forced. *The two arguments are deliberately independent*: a companion result on the causal occurrence of gravitational collapse stands on causal structure alone and needs none of the cosmological evidence below [5], and nothing in the present argument draws on it. That they meet is therefore evidence and not construction. We stress the register at the outset, as the companion papers do: nothing here modifies general relativity. The equations are unchanged. What is established is which of the formally available foliations is the physical one—and, read ontologically (§4, Theorem 1), that this physical foliation is the *lapse*, the objective cosmic present, while the relativity of synchrony is the *shift*: the measured isotropy thereby settles the century-old question the relativity of simultaneity left open—synchrony relative, simultaneity absolute—exposing the reading that denied an objective present as a modal fallacy, and fixing this reading as the necessary and sufficient augmentation of general relativity for a coherent description of an existing, evolving world.

2 The two contributions to the observed anisotropy

The redshift along a line of sight is the total expansion the photon underwent, $1+z = a_{\rm obs}/a_{\rm em}$, equivalently $\ln(1+z) = \int H\,dt$ integrated along the ray: the cosmological redshift simply *is* the accumulated expansion in that direction, a path integral and not a property of the source. *That path integral is a conserved quantity written out, which is why it is a property of the path.* In conformal form $ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2)$ the vector ∂_η is a *conformal* Killing vector, so its charge $\xi\cdot p$ is conserved along a geodesic only where $p\cdot p = 0$; on the null cone that charge is $a\omega$, and its constancy *is* $1+z = a_{\rm obs}/a_{\rm em}$, whose logarithm is $\int H\,dt$. *The restriction to the null cone is not incidental to the argument below but the reason it works*: for a massive carrier $p{\cdot}p = -m^2$, the charge is not conserved, and no such line-of-sight integral exists—so the redshift is a statement about light and could not have been obtained from any other messenger[L14]. The observed temperature is $T_{\rm obs}(\hat{n}) = T_{\rm em}(\hat{n})/(1+z(\hat{n}))$, so the observed anisotropy separates exactly into two physically distinct contributions,

$$\frac{\delta T_{\rm obs}}{T_{\rm obs}} = \frac{\delta T_{\rm em}}{T_{\rm em}} \;-\; \delta\!\int H\,dt, \tag{1}$$

a *source* term fixed by conditions on the last-scattering surface, and a *cumulative* term set by the directional variation of the integrated expansion rate over the whole path. The two are independent: the value of one constrains the other not at all. Observed isotropy therefore requires *both* to be small—uniformity at decoupling *and* uniformity of the subsequent expansion—and these are precisely the two kinematical assumptions the standard model installs by hand, homogeneity at decoupling zeroing the first term and a single direction-independent scale factor $a(t)$ zeroing the second.

The decisive consequence is seen by granting the common reading its strongest form. Suppose the last-scattering surface were perfectly homogeneous, $\delta T_{\rm em} = 0$ exactly. Then (1) reads $\delta T_{\rm obs}/T_{\rm obs} = -\,\delta\int H\,dt$, with nothing left over: the entire observed isotropy is the statement that the integrated expansion was the same in every direction. Homogeneity at the source does not *supply* the isotropy; granted perfectly, it removes one term and leaves the observed isotropy as a *direct measurement of uniform expansion.* The widespread supposition that the monopole's isotropy is delivered by homogeneity at last scattering, full stop, is thus a category error—it credits the source for a fact about the *observed* radiation that resides entirely in the intervening expansion. What the remainder of this paper does is put a number to the cumulative term under the one coherent alternative to a global $a(t)$, and read that number against the data.

3 The redshift-isotropy floor

3.1 The hypothesis under test

The hypothesis the floor tests is that the large-scale expansion rate tracks the matter lumpiness region by region—that the expansion follows the local density, with no single global scale factor. This is the operative reading of the backreaction and timescape programmes [13, 14]; we make it quantitative through the separate-universe correspondence, in which a spherical region of uniform overdensity evolves as a closed Friedmann patch.

To linear order the separate-universe correspondence assigns a region of fractional matter contrast $\delta \equiv \delta\rho/\rho$ a fractional deficit in its local scale factor. Matter conservation gives $\rho \propto a^{-3}$, hence $d\rho/\rho = -3\,da/a$, so

$$\frac{\delta a}{a} = -\frac{1}{3}\,\delta. \tag{2}$$

An overdense region has expanded less; an underdense region more.

3.2 Accumulation along the line of sight

The observed CMB temperature in a given direction is $T_{\rm obs} = T_{\rm lss}/(1+z)$ with $1+z = a_{\rm obs}/a_{\rm em}$ the total expansion factor along that line of sight, so to the order kept $|\delta T/T| = |\delta a/a|$: the temperature contrast between directions is the fractional contrast in accumulated expansion. A CMB photon traverses many regions of differing density; modelling its path as a tube binned into N statistically independent cells of comoving size $R = 8\,h^{-1}\,$Mpc, each carrying rms contrast σ_8, the accumulated fractional redshift contrast is the path *mean* of the per-cell contrasts. By the central limit theorem its standard deviation is

$$\sigma_{\rm path} = \frac{\sigma_{8,\rm eff}}{3\sqrt{N}}, \tag{3}$$

the factor 1/3 from (2) and the $1/\sqrt{N}$ the averaging-down of independent contributions—a path mean, not a sum.

3.3 The number, and why it is a floor

The effective contrast weights σ_8 by its linear growth along the path, $\sigma_{8,\rm eff} = \int \sigma_8(z)\,w(z)\,dz / \int w(z)\,dz$ with $\sigma_8(z) = \sigma_8(0)\,D(z)$ and $w = H^{-1}$; for $\Omega_m = 0.315$, $\sigma_8(0) = 0.8$ and $z_{\rm lss} = 1089$ [12] this gives $\sigma_{8,\rm eff} \approx 0.285$. The comoving distance to last scattering is $d_{\rm lss} = (c/H_0)\int_0^{z_{\rm lss}} dz/E(z) \approx 9.4\,h^{-1}\,$Gpc, so $N = d_{\rm lss}/R \approx 1174$. Equation (3) then gives

$$\sigma_{\rm path} \approx 2.8 \times 10^{-3}. \tag{4}$$

Every choice in this estimate biases it *downward*, so the result is a genuine floor:

- the full matter coefficient 1/3 is retained (dropping it would treble the estimate);
- growth-weighting suppresses σ_8 at high redshift ($\sigma_{8,\rm eff} = 0.285$ against $\sigma_8(0) = 0.8$), where a constant σ_8 along the path would give an estimate $\sim 2.8\times$ larger;
- treating the cells as independent *maximizes* N: gravitational clustering produces positive correlations along the line of sight, which can only increase the variance of the path mean, so the no-correlation value is a strict lower bound (coherence over basins of attraction at $\sim 138\,h^{-1}\,$Mpc would lower the effective N to ~ 68 and raise the estimate $\sim 4\times$; the increase is a theorem rather than a caution, since for N cells of average pairwise correlation $\bar\rho$ the variance of the mean is $(\sigma^2/N)(1+(N-1)\bar\rho)$, which exceeds the independent value for every $\bar\rho > 0$ and falls below it only for $\bar\rho < 0$[P4R3L24]);
- the realized void/wall density contrast on the scales the programmes invoke is of order unity, well above σ_8 at $8\,h^{-1}\,$Mpc;
- and the $1/\sqrt{N}$ of (3) is only the *white-noise limit*[L12] of the path average, which biases the estimate downward through a channel none of the items above covers. A path mean is a window in Fourier space, with variance $\int dk\,P(k)\,|W(k)|^2$ and $|W|^2 = \mathrm{sinc}^2(kL/2)$ for a

tube of length L. The $1/\sqrt{N}$ of (3) is exact only for $k \gg 1/L$: at $k = 1/L$ the window is still 0.92, at $3/L$ it is 0.44, and only by $10/L$ has it fallen to 0.04. *Modes longer than the path are not averaged down at all*—they contribute coherently along its whole length, and coherent contributions add to the scatter rather than cancelling in it, so the true floor is higher than (4) on this count as well. *That last statement carries a boundary, and it is worth drawing because the quantity being bounded is an anisotropy rather than a single path's variance*: $\mathrm{sinc}^2(kL/2)$ is the window for the variance of one path, whereas every sightline is cast from the same observer, so a mode much longer than the observable region takes one value over the whole of it, lands on every direction alike, and is removed with the monopole. The coherence invoked here therefore adds to the scatter for $k \gtrsim 1/L$ and cancels out of it below—measured, the across-sky scatter tracks the single-path variance to 1% at $kL \gg 1$ and falls to 2×10^{-4} of it at $kL \ll 1$. *The boundary costs nothing here*: on a CDM spectrum the fraction of the path-mean variance lying below $k = 1/L$ is 3×10^{-6}, and 95% of it sits above $kL \approx 70$, so this bullet's contribution is downward everywhere the variance actually lives[P4R4L24].

3.4 The floor against the data

The monopole redshift is isotropic—after removal of the dipole, Galactic foregrounds, and the standard secondary anisotropies—to $\lesssim 3 \times 10^{-6}$ [12].[P4R1] The floor (4) exceeds this by some three orders of magnitude. *The comparison is a lower bound against an upper limit, which is what makes it robust rather than close*: every free choice in the estimate pushes the two further apart, so the exclusion cannot be weakened by a different estimate, only by a different physical claim. The margin is a factor of 923[L25], and it is worth stating what would be required to close it—the observed isotropy would have to be 9× worse than measured for the exclusion to fall to two orders of magnitude, 92× worse for one, and 923× worse for it to vanish. The hypothesis that the expansion rate is locally set by the matter distribution is therefore excluded, and not marginally.

It is essential that this is a distinct effect from the ordinary Sachs–Wolfe and integrated Sachs–Wolfe anisotropies [25], not a re-derivation of them. In the uniform-expansion picture the matter lumpiness is clustering upon a single background $a(t)$, carried by a metric potential Φ; a photon's gravitational redshift descending into a potential well is undone climbing out, so the contributions *telescope* along the path, leaving the endpoint potential ($\sim 10^{-5}$) and the small integrated term, with no $\sqrt{N}$ accumulation. The floor computed here is the contrary case—a genuine differential *expansion*, a cosmological redshift that accumulates because there is nothing to cancel against. The observed anisotropy ($\sim 10^{-5}$ in total, $\lesssim 3 \times 10^{-6}$ once secondaries are removed) thus selects the telescoping, uniform-expansion picture over the accumulating, differential one. The isotropy of the monopole is not accounted for by large-scale statistical homogeneity—that homogeneity is fully consistent with the differential-expansion hypothesis, which would still produce the $\sim 10^{-3}$ scatter of (4)—but by uniform expansion in a cosmic time.

A differential-expansion programme can evade the floor only by requiring the local rates to average to a common value in every direction: a uniform large-scale Hubble flow. But that requirement is itself a global uniform expansion in a cosmic time, the very structure such programmes set out to dispense with; imposing it to recover the observed isotropy reinstates the cosmic rest frame and the objective simultaneity it carries. Either way, the observed isotropy of the monopole redshift forces a uniform cosmic expansion.

3.5 Uniform expansion, lumpy matter

The floor constrains the expansion *rate*, not the matter content. CR's real space is locally curved and inhomogeneous—the matter density is the bend of the spatial cut off the de Sitter vacuum profile [7, 8]—and that lumpiness is untouched by the argument. What is excluded is an inhomogeneous expansion rate *tracking* the lumpiness: the layer advances uniformly in cosmic time while its intrinsic geometry carries the bend. Uniform expansion and lumpy matter are consistent, and only the former is at issue here.

4 What the exclusion establishes

The floor closes the statistical escape, and one further escape is logically available and must be closed separately. A finely tuned, spherically symmetric inhomogeneity centred on the observer would produce isotropic redshift about us with no global uniform expansion; being structured rather than statistical, it is not touched by the $\sqrt{N}$ floor. It is closed instead by the Copernican principle together with the independent isotropy of the *expansion history* itself: standard rulers (baryon acoustic oscillations [29]) and standard candles (Type Ia supernovae [30, 31]) reconstruct the expansion along many directions and agree across the sky, so the uniformity is not the vantage artefact of a privileged centre [26, 27, 28]. With the statistical alternative excluded by the floor and the centred one by the Copernican step, the disjunction is exhausted: the observed isotropy forces uniform cosmic expansion.

What that exclusion reaches is the point of the paper. The region the data reject is not a parameter corner but the entire class of histories lacking a single global scale factor on a single global time; to reject that class is to select a global, time-ordered expansion, which is the cosmic foliation made dynamical. And the foliation is logically prior to everything the standard model otherwise assumes—"space" is one of its slices, "expansion" an ordered family of them, "isotropy" and "homogeneity" properties of a slice—none of which is even statable until the foliation is in hand. So a single datum establishes, from the bottom up, the cosmic time on which the rest stands, the uniformity of its advance, and—within the observable region and under the Copernican principle—the maximal symmetry of its slices. The model's foundational kinematics are not idealizations the data tolerate; they are what the data force, the one coherent alternative excluded.

5 The history the measurement closes

The structure this datum forces was posited, attacked, defended, and abandoned within four years, and the record is worth setting down because the abandonment shaped a century of practice.

Einstein's cosmological paper of February 1917 assumed, in his own words and marked as such, "against the spirit of relativity", that *"there is a system of reference relatively to which matter may be looked upon as being permanently at rest"* [22]. *His ground was empirical*: "the most important fact that we draw from experience as to the distribution of matter is that the relative velocities of the stars are very small as compared with the velocity of light". In a special-relativistic world one would expect neighbouring bodies at velocities drawn uniformly from $(0, c)$; the observed smallness of proper motions is evidence that the world lines are nearly parallel, and the three-space in which they are at rest is distinguished. *That inference is the one this paper's floor now closes at three orders of magnitude*, and it was drawn from stellar velocities a century before.

De Sitter's reply came one month later and countered every consideration in turn [1]. The boundary conditions were dismissed as unobservable in principle and, given the new λ-term, achievable anyway; the world-matter Einstein thought essential was declared to "serve no other purpose than to enable us to suppose it not to exist"; and the choice among the candidate universes was placed outside physics altogether—it "cannot be decided by physical arguments, but must depend on metaphysical or philosophical considerations, in which of course also personal judgment or predilections will have the same influence". *And the criticism he pressed most was the one that matters here*: that in Einstein's solution "time has a separate position" from space.

Three years later the same structure was defended on geometric grounds, by the relativist best placed to do it. Eddington held that "the world taken as a whole has one direction in which it is not curved; that direction gives a kind of absolute time distinct from space", that relativity "is not concerned to deny the possibility of an absolute time, but to deny that it is concerned in any experimental knowledge yet found", and that "it need not perturb us if the conception of absolute time turns up in a new form in a theory of phenomena on a cosmical scale, *as to which no experimental knowledge is yet available*" [17]. The time in question is that of "a being coextensive with the world", which is the foliation. *He names the absence of the discriminator, and selects anyway.*

What followed was silence. The 1917 paper was the last in which Einstein published the details of his cosmological considerations. After Hubble's result he wrote some five pages on the problem in two short articles, the second co-authored with de Sitter, while de Sitter published volumes; and his later rejection of the cosmological term shaped the expansion paradigm for the rest of the century [2]. *He never returned to the question his own assumption raised*—how a cosmic time distinguished by the matter's rest frame stands to the relativity of simultaneity—and so far as the record shows never addressed it in print.

The measurement reported above closes that arc. What Einstein posited apologetically on stellar velocities, what de Sitter placed beyond physical argument, and what Eddington defended without evidence, is what the redshift isotropy now forces. *The assumption marked "against the spirit of relativity" was correct*, and the objection that it gave time "a separate position" was correct too—as a description of the structure, though not as a reason to reject it. *We record the history because the reasoning that abandoned this structure is still in the field's hands*, and because a selection made in 1920 in the declared absence of evidence, met by a measurement in the decades after, is the pattern by which theory-choice earns its standing [6]. *And the parallax of this paper's title should be read with the same care the historical case requires.* Stellar parallax went undetected for two millennia and was read as telling against the Earth's motion, but before the cosmos was given depth the relevant parallax was not a conceivable measurement at all—the stars sat on a sphere and had no distances to be displaced against [6]. The objection is post-Digges, and it is Tycho who presses it. *The lesson is the one this paper's floor turns on*: an absent discriminator is not an absent fact, and whether a discriminator is even available is a question about the picture in hand rather than about the world.

What the standard model installs as assumptions, the present framework derives. The cosmic-time variable was put in by hand at the outset—assumed in Einstein's 1917 cosmology [22] and carried through Friedmann's expanding solutions [23] to the present model, where a single global scale factor on a single global time is the background fixed before any matter is placed upon it. That foundational structure is here forced by measurement. *A second assumption entered in the same 1917 paper, and it runs in the opposite direction from what the present result establishes.* Einstein's static solution fixes the cosmic scale from the content: with $\lambda = \kappa\rho/2$ the radius is $R = 1/\sqrt{\lambda}$, so *the density of the world-matter determines the size of the world* [22]—and Friedmann's equations carry that direction of determination forward, the scale factor's evolution being sourced by what the universe contains [23]. What the isotropy forces is the reverse ordering for the *rate*: it is fixed independently of how the content is distributed, since a rate tracking the content is precisely what the measurement excludes. Whether the same reversal extends from the rate to the scale is not settled here and is taken up in a companion [3]. This is the empirical motivation the title names: not a modification of general relativity, whose equations are untouched, but the measured selection of the foliation the theory admits and does not otherwise single out.

What the measurement forces is the empirical half of a structural result, and the half in hand fixes the other, so it is worth stating whole. General relativity supplies a four-geometry and field equations but does not single out which of the foliations its formalism admits is the physical one—as the companion papers stress and as the present forcing makes vivid [3]. The standard interpretation fills that silence by collapsing two distinctions at once: it reads the relativity of simultaneity as the absence of any objective present, and it reifies the four-manifold as a thing that exists. Each is a category error of the same family as the one corrected in §2—there, crediting the source for a fact that resides in the intervening expansion; here, crediting the manifold for what the evolving world does. Pried apart, the two are the parts a coherent formal description of an existing, evolving world requires; and they are exactly the lapse and the shift of the 3+1 split read ontologically—the Newtonian part, an absolute rate of advance, and the Einsteinian part, the relativity of synchrony, the formalism the standard reading had fused and a coherent description must hold apart.

The relativity of *synchrony* is general—a feature of relative motion, the light postulate fixing only its magnitude [10, 33]—and it is the shift, the freedom of a moving observer to tilt the slices off the comoving normal. *Simultaneity* is the separate question, and to read the relativity of synchrony as the denial of an objective simultaneity is a modal fallacy, the absence of a local test mistaken for the absence of a fact—the imperative's characteristic dual, established as such in the companion epistemic paper [6, 10]. The lapse is the other part: the rate of advance of the layer that exists, what the clock measures [34]. To deny it—to hold the four-manifold itself the existent—is to treat occurrences as existents, which on analysis smuggles a fifth, meta-temporal dimension along which the block is supposed to "exist," and is incoherent as a basic description [34, 32]. The two corrections are not optional glosses on the formalism; they are what makes "a world that exists and evolves" a statable object at all. With them restored the measured result binds: the redshift isotropy forces the lapse—a global, uniform rate of advance in a cosmic time—while the relativity of synchrony, untouched, is the shift, the existent the evolving layer the foliation orders and the manifold its record.

Theorem 1 (The necessary and sufficient augmentation)**.** *Augmenting general relativity by fixing a physical foliation and reading it ontologically—the lapse the objective rate at which the existing world advances, the shift the relativity of synchrony—is both necessary and sufficient for a coherent formal description of an existing, evolving world.* Necessary: *the two parts are the pried-apart forms of the existence/occurrence and synchrony/simultaneity distinctions, and a description lacking the lapse part collapses existence into occurrence—the block reading, incoherent as a basic description.* Sufficient: *the two parts close the only coherent escapes—the "events exist" horn to the existence/occurrence correction as a category error, the "no objective present" horn to the synchrony/simultaneity correction as a modal fallacy, the latter then falsified outright by the measured isotropy—leaving the layered, cosmically-timed description with no surviving alternative. The augmentation alters none of general relativity's equations; it fixes which of the foliations they admit is physical and reads that one as the existent.*

The measurement is what turns this from a coherent option into a forced one: the lapse the theorem requires is not posited but measured, the cosmic foliation read directly off the redshift isotropy. The augmentation is thus not merely the structure a coherent description *may* take but the one the world is found to realize. What the theorem does *not* fix is which geometry realizes the augmented structure. It fixes that the structure is necessary and sufficient, and the measurement fixes that its necessary half obtains; which four-geometry carries it, and how matter sits on that geometry, are not settled here and are taken up in companions [3, 7, 8]. What is settled *here* is the structural verdict with its necessary half measured: the two-part augmentation—the completion of the Newton–Einstein formalism the standard reading left fused—is the necessary and sufficient one.

The warrant for the theorem's two moves is worth marking, because both are ordinary scientific inference rather than appeals to taste. Diagnosing the "events exist" horn as a category error is a correction of what the formalism actually asserts, not a preference about it. And preferring—as the floor above and the assessment below both do—the description that *requires* the observed structure over one that merely *permits* it is inference to the best explanation, the same step by which any theory that predicts a datum is preferred to one that accommodates it. Neither move is peculiar to this paper, and neither is taken on authority: each is made here on its own, and a companion paper takes up the general question of when such inferences are reliable [6]. *Nothing below draws on that treatment*, which is the point—an instance reached without the general account is a case that account can later rest on without circularity; and the category-error correction of §2 is the same method's first turn within this paper, so the paper's measured core and its structural verdict are operations of one grounded discipline.

It remains to weigh, against this result, the one objection that the uniform expansion is justified rather than assumed. It is held that Green and Wald have shown small-scale inhomogeneities to backreact negligibly on the large-scale metric, so that the Friedmann model is self-consistent with the averaged matter as its source [15], against the contrary case that backreaction is dynamically significant [13, 14]. Kept apart from the theorem and then set against it, the demonstration is seen to take as its input precisely what the floor forces: it assumes the metric already close to Friedmann–Lemaître–Robertson–Walker form—the cosmic time, the single global expansion—and proves the lumpiness does not disturb it. That is a statement about the *stability* of the assumed foliation, presupposing it and establishing its robustness, not deriving it—what is small in the demonstration is the metric perturbation ($\sim 10^{-5}$), while the matter contrast it rides on is nonlinear, so the licence to treat the configuration as a perturbed Friedmann model is itself a statement about the assumed background. The floor owes none of this, forcing the cosmic time and the uniformity of its advance from the redshift isotropy directly, a datum prior to any slice, with no Friedmann structure assumed in its derivation. This settles the standing dispute structurally and from outside it: the uniform expansion that one side assumes and the other questions is forced by measurement whichever way the averaging argument is adjudicated, so the most rigorous defence of the standard treatment is found to presuppose the very structure the measurement supplies, and the backreaction question, however it resolves dynamically, cannot reach the foliation the floor has fixed. Were the backreaction case to carry genuine large-scale content, it would enter on that merit and against this result; it does not enter as the ground of the uniform expansion, which the measurement has removed from its reach.

The scope is worth marking. This contradicts none of the standard model's fitted predictions; with its assumptions in place it reproduces the expansion history observed, which is why it stands. What the measurement changes is the standing of those assumptions: the cosmic time and the uniformity of its advance are forced from the bottom up, prior to and independent of the dynamical fit, not conventions adopted for it. What the measurement does *not* do is install the replacement—the positive account, that the expansion is set by the cosmological constant alone with the matter density a reading of cosmic epoch [3], is motivated by this forcing but not compelled by it, and is left to the companion framework rather than asserted here.

This is the empirical counterpart, made quantitative, of the conceptual point that the cosmic present is measured rather than posited. As stellar parallax once converted the Earth's motion from an interpretive option into a measured fact, the isotropy of the cosmological redshift—with our own velocity through the rest frame read directly from the dipole, (369.82 ± 0.11) km/s toward Crater [12]—converts the cosmic foliation from a modelling convenience into a measured feature of the world. The general result that no exterior temporal slicing can record a finite-time completion of gravitational collapse [5] stands on causal structure alone and needs none of this evidence; what the evidence adds is that the slicing that result concerns is not one option among many but the one the universe has let us measure. *Scope.* The temporal claim—uniform expansion throughout cosmic history—is clean, being the integral from decoupling to the present along the observed rays; the extension to a globally maximally symmetric space rides on the Copernican principle and is an extrapolation beyond the directly constrained region. *And the scope of what is claimed is worth stating against what is not.* What the isotropy measures is the uniformity of the expansion rate along the observed rays; it does not measure, and this paper does not claim, anything about which geometry carries that expansion or how the matter within it is arranged. The matter lumpiness is untouched. What is excluded is an inhomogeneous expansion *rate*, and the reason the two are separable is the reason the floor exists at all.

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Appendix R Computational Receipts

Each computation marked R in the text is verified by a runnable receipt in the `receipts/` directory that ships with this work; status OK = verified (traced, run, output matches the claim). This appendix is generated from the receipt index, not maintained by hand.

P4R1 `P04_redshift_isotropy_floor` [OK]
sec:floor (redshift-isotropy floor sigma_path=sigma_{8,eff}/(3 sqrt N)˜2.8e-3 exceeds observed monopole isotropy (¡˜3e-6) by ˜3 orders -¿ matter-tracking expansion excluded). *Computes:* d_lss integral -¿ N=1174; floor 2.77e-3; exclusion ratio 924˜10ˆ3; genuine-floor (biases down) + delta a/a=-1/3 delta derived *Bound:* linear separate-universe deficit taken as input *Run:* `python3 receipts/P04_modern_parallax/P04_redshift_isotropy_floor.py`

P4R2 `I20_the_redshift_is_a_conformal_killing_charge_on_the_null_cone` [OK]
sec:decomp (I20 — THE REDSHIFT IS A CONFORMAL-KILLING CHARGE ON THE NULL CONE. Integrable-systems field bake, probe I20. In conformal FLRW form d/deta is shown by explicit Lie derivative to be a CONFORMAL and not an exact Killing vector, so its charge is conserved only where p.p = 0; there it is a*omega, whose constancy IS 1+z = a_obs/a_em and whose logarithm is int H dt. Carries a MASSIVE CONTROL: the spatial momentum still falls as 1/a but a*E is not constant for m nonzero, and it recovers constancy in the massless limit – so the null-cone restriction is load-bearing and P04's line-of-sight integral is a statement about light.). *Computes:* runs clean; nine verdicts including a massive control that fails off the cone and recovers in the limit *Bound:* derived here; no parameter pinned *Run:* `python3 receipts/P04_modern_parallax/I20_the_redshift_is_a_conformal_killing_charge_on_the_null_cone.py`

P4R3 `R2_the_papers_correlation_figures_are_the_ones_nothing_checked` [OK]
sec:floor (R2 — P04's CORRELATION FIGURES ARE CORRECT AND ITS RECEIPT CHECKED A PLACEHOLDER INSTEAD. Probability field bake, probe R2, and it exists only because of the step the six-field campaign skipped. P04's probability vocabulary is near zero — probability x0, stochastic x0, random x0, variance x2 word-bounded — so the term side scored it CHECKED-NEGATIVE. Read from contents it is the highest paper in the field: its whole argument is the standard deviation of a path-averaged random field. The read found that P04's two quantitative correlation figures are right — N_eff = d_lss/138 = 68 and the sqrt(N/N_eff) = 4.2x rise — while its existing receipt checks the caveat with a placeholder N/4 = 293 giving 2x, so the direction was checked and the stated magnitude never was. Also verifies all three of P04's window values, |W|ˆ2 = sincˆ2(kL/2) at k = 1/L, 3/L, 10/L, as 0.92, 0.44 and 0.04. Verdict 4 turns 'positive correlations can only increase the variance' into a theorem: Var(mean) = (sigmaˆ2/N)(1+(N-1)rhobar), symbolically and by simulation. CONTROL: a NEGATIVE rhobar must reduce the variance, or the positivity clause would be vacuous.). *Computes:* runs clean; six verdicts and a sign control *Bound:* derived here; the probe that overturned a CHECKED-NEGATIVE verdict, and the ledger's claim that P04 'returned empty on all six fields' is corrected in place *Run:* `python3 receipts/P04_modern_parallax/R2_the_papers_correlation_figures_are_the_ones_nothing_checked.py`

P4R4 `R50_the_long_wavelength_bullet_reverses_for_the_observable_and_the_corner_is_three_parts_per_mill`
[OK]
sec:floor (P04's fifth 'biases downward' bullet says modes longer than the path contribute coherently and so add to the scatter; the quantity being bounded is an ANISOTROPY and every sightline is cast from the same observer, so a mode longer than the observable region is a monopole and cancels out of it). *Computes:* casts sightlines from one origin through a plane-wave Gaussian field and measures BOTH the single-path variance and the across-sky scatter: the ratio is 0.990 at kL in 100..1000 and 2.0e-4 at kL in 0.01..0.1, monotone between, stable under halving the modes, doubling the sightlines and changing the seed; then integrates a BBKS CDM spectrum against the paper's own d_lss and finds only 3.1e-6 of the path-mean variance below k = 1/L, with 95 percent of it above kL = 70 *Bound:* SUPPORTS P04 – the bullet needs one clause, not a retraction, and the clause is landed. NOT CLAIMED: any value of the floor itself (P04_redshift_isotropy_floor owns that); the other four bullets and R2's along-path correlation theorem are untouched *Run:* `python3 receipts/P04_modern_parallax/R50_the_long_wavelength_bullet_reverses_for_the_observable_and_the_cor`

`B11_three_triples_two_identifications` [OK]
sec:cycloid (THREE three-fold structures and TWO open identifications carried in two registers that do not reference each other — the root triple is the shared vertex of both and appears

in neither as such). *Computes:* asserts all three structures at source, P4s two-triples question and its {0,1,2} requirement, and that neither register cites the other (8 checks) *Bound:* NOT-A-PAPER-CLAIM — connects two open items; no identification established *Run:* `python3 receipts/L221_the_bridge/B11_three_triples_two_identifications.py`

`B12_the_two_triples_are_typed_differently` [OK]

sec:cycloid (the two triples are NOT one structure: the ROOT triple is the f=0 locus (roots 1/sqrt3, 1/sqrt3, -2/sqrt3, sum zero) while the CAUSAL triple is three VALUES of f each with one locus — three-element sets of different KINDS). *Computes:* asserts P4s question and its three-equally-spaced-values clause, computes the Nariai cubics roots and zero-sum and double root, matches PO-2s two distinguished roots, and counts the loci per level (8 checks) *Bound:* NOT-A-PAPER-CLAIM — a NEGATIVE result discharging the corpuss only DISCOVERABLE-PROOF *Run:* `python3 receipts/L221_the_bridge/B12_the_two_triples_are_typed_differently.py`

Appendix L The Knowledge Ledgers

Each claim marked $^{\mathsf{L}}$ in the text rests on a field bake or the figure–theorem bake recorded in the named ledger, which ships with this work and carries the probes, the receipts, and the three registers kept apart — what bit, what bounced, and where the boundary is. A ledger is working apparatus, not a publication, and is cited here rather than in the bibliography. This appendix is generated from the ledger index, not maintained by hand.

L12 `HARMONIC_ANALYSIS_LEDGER.md` [field bake]
The harmonic-analysis field-bake ledger — what bit, what bounced, and the boundary. Second of the four fields 'L-272''s re-survey left outstanding. 'OWED' 622

L25 `STATISTICS_INFERENCE_LEDGER.md` [field bake]
The statistics/inference field-bake ledger — what bit, what bounced, and why. The last unbaked field in 'THE_MATHEMATICS_REACH''s candidate set, and the one 'A5.5' needs. 'OWED' 622

L24 `PROBABILITY_LEDGER.md` [field bake]
The probability and stochastic-processes field-bake ledger — what bit, what bounced, and where the boundary is

L14 `INTEGRABLE_SYSTEMS_LEDGER.md` [field bake]
The integrable-systems field-bake ledger — what bit, what bounced, and where the boundary is