The fermion sector of Cosmological Relativity: three chiral generations from the maximally-symmetric slicing structure

Daryl Janzen

Abstract

The companion papers establish that the black-hole/de Sitter substrate of Cosmological Relativity (CR) is a maximally symmetric manifold sliced perspectivally, and that the Standard Model gauge group does *not* arise as a continuous isometry of it: the geometric route to $\mathfrak{su}(3)$ is excluded, leaving the substrate's discrete orientation structure as the one opening for matter.

This paper puts a Dirac spinor field on the slicing structure and asks what that discrete opening delivers.

Three results follow, each stated at the weight it is earned.

First, the signed areal radius forces the radial Dirac superpotential to change sign at the throat, binding exactly one chiral zero-mode there; its chirality is the diagram-automorphism parity $R = \gamma^5$ (an exact solution, not an assertion). The norm in which it binds is the one CR's ontology selects, and the point is load-bearing[P14R9]: the fermion is a mode of the *existent spatial leaf*, of which the spacetime is a projection, so its measure is the induced proper distance $d\ell = dr/\sqrt{|f|}$, in which the horizon turning points lie at *finite* distance and the $r = 0$ crossing is an integrable square-root singularity.

In the conserved *spacetime* Dirac norm—the tortoise measure—the same static mode does *not* normalize, the horizons standing infinitely far.

The two are not interchangeable, and CR reads the fermion on the leaf, where it is a bound state.

Second, the substrate is symmetric in three 120°-separated hinges, and the maximally-symmetric—least-arbitrary—matter construction, which is the one CR's own founding principle selects, places a slicing plane on each: three throat walls, hence three chiral zero-modes. A one-hinge truncation is excluded not as disfavoured but as carrying an unfixed arbitrary modulus, which the principle forbids. That count is a well-defined index in the precise sense the ontology makes available: in the leaf's proper measure the closed slicing has *finite* total length[P14R10], so the leaf is compact and its Dirac operator carries a well-defined analytical index—exactly where the bulk index on the non-compact substrate, which the boundary paper's non-compactness escape turns on, is obstructed. *And the finiteness survives the member this programme's cosmology actually is*: at the Nariai double root the two horizons merge, f acquires a double zero, and $1/\sqrt{|f|}$ becomes a simple pole—the non-integrable exponent, so the hypothesis fails *pointwise* exactly there. The length nonetheless converges, to $1.8138\,\alpha$ as the gap closes through four decades, because $\int \mathrm{d}r/\sqrt{(r-r_b)(r_c-r)} = \pi$ independently of the gap: *the interval closing compensates the integrand blowing up*, so the pointwise divergence never reaches the limit[P14R10L10].

The signed-radius flip moreover passes through the $r = 0$ *branch point*[P14R49]—the wall and the branch point being one locus with two properties rather than two loci that coincide, which is checked on locus, type, definition, equivariance and separability—rather than a single-valued crossing on a loop, so the even-crossing constraint does not apply and the three same-chirality modes, on disjoint support, have no cancelling partner: $\dim\ker_+ = 3$, $\dim\ker_- = 0$, the net chirality a γ^5-graded index.

Its stability under deformations preserving the three-wall structure is the expected behaviour of such a count and is stated here at traced weight: the receipt establishes the mechanism—one mode per wall, the even-crossing obstruction on a simple loop, and the necessity of the $r = 0$ branch point—and marks the Atiyah–Singer statement on the branched bead as traced rather than computed.

Third, the three zero-modes are permuted by the full S_3 generated by the three hinges, and that S_3 is not a second group resembling the substrate's own: each hinge designates one of the three roots of the horizon cubic as its own black-hole horizon, so a transposition of roots is a hop to a neighbouring hinge and the Weyl S_3 *is* the relation among the three hinges. Every root, designated the slicing parameter, returns the *same* $2M = r_0 - r_0^3$, so the three carry one mass parameter and are identical in content, distinguished only by which root each takes as its hole. *Which index that threeness is, this paper settles* (Section 6.1): the hinge S_3 is a *within-state* index and not a family symmetry, the generations' own threeness being the turnaround's deck $\mathbb{Z}_3$, with the wall structure fixing the *number* at either seat.

So the wall structure delivers a count and a chirality rather than a seat: a generation is not a wall, and the wall fixes how many sit at whichever seat carries them.

Those two data—a chirality and a count—are the two factors of one group, $D_6 = S_3 \times \mathbb{Z}_2$, and the paper closes by saying why they descend differently, which is a reading of established structure rather than a fourth result.

The factors act on independent structures, the Weyl S_3 on the three roots and the inversion on the two rulings; on the substrate's waist those are the two relations a figure can bear to a circle, the roots being the three special points *on* it and the rulings the lines *tangent* to it, and *on* and *tangent* are independent relations. *The directness of the product is not itself the evidence for that, and should not be cited as though it were*: $\mathrm{Aut}(A_2)$ is a Weyl S_3 extended by a diagram $\mathbb{Z}_2$, every automorphism of S_3 is inner, and a semidirect product by an inner automorphism is isomorphic to the direct one—so no semidirect alternative exists to be ruled out. What the two relations supply is which factor is which, and that they differ in *kind*.

The kinds then differ for the same reason: a tangent is a line of the substrate, so exchanging the two rulings is a motion of it—an isometry, acting on the cut spinor as γ^5, and chirality descends *gauged*; a root labels a different cut, so permuting the three moves through the solution space—the monodromy symmetry, and flavour is *global*. The Standard Model's arrangement of a gauged chirality against a global flavour is, in this reading, the difference between being *on* the one circle and being *tangent* to it, and this sector is where that difference becomes two physical data.

The net structure—a count of three, a chirality, and a within-state S_3—is the Standard Model's *discrete flavour structure*, obtained here as the maximally-symmetric slicing of a maximally-symmetric substrate.

It is not the mass spectrum (electroweak breaking), and the boundary is drawn explicitly; what it does deliver about the gauge sector is the fourth result below. *The count that correspondence has to meet is specified rather than gestured at*[P14R21]: one generation splits 12 coloured against 3 colourless (4 with a right-handed neutrino), and the sector's deliverables are matched factor by factor against that split rather than against the total. *Where that matching succeeds and where it fails can now be stated exactly. The twelve factor as $3 \times 2 \times 2$—colour, the horn, and the ruling—giving four classes, against the Standard Model's five multiplets per generation, and the shortfall is precisely one pair, on the right-handed side, where $SU(2)_L$ does not act and u^c and d^c are separate singlets. The cause is identified rather than left as a discrepancy: the grading operators available here cannot produce it, since the species operator acts identically on both chirality eigenspaces and one-dimensional characters cannot act on one and not the other; and the reason they cannot is that the geometry constructed is the polarised, and therefore achiral, member of its own range.* That cause is a theorem rather than a diagnosis[7]*: where the handedness-exchanging involution is a realised symmetry it commutes with the horn swap, so the swap's two chirality restrictions are conjugate and cannot differ in orbit structure—and the missing multiplet is exactly such a difference, a doublet against two inequivalent singlets. The shortfall is therefore forced and not merely found, and it costs more than a multiplet: the same* *argument makes the sector's isospin structure vector-like, on which the anomaly conditions are automatic and fix no hypercharge. The unpolarised member—which carries the second propagating mode, and with it handedness—is built in the companion development, where the exchanging map is shown to lie outside the identity component and to carry a conserved twist whose sign it reverses, with an explicit inhomogeneous propagating member of it carrying that handedness as the definite-signed winding of its turning polarisation plane [15]; what is not built is the Dirac sector on it.* So the two absences are not one*: the polarisation is supplied and the multiplet is not, and what stands between them is a computation on the chiral member rather than a geometry. This sector, built on the polarised cut, matches the Standard Model's left-handed shape and draws on the right-handed pair a distinction the Standard Model does not: its horn orbits are* $2{+}2$ *where the observed content is* $2{+}1{+}1$*. The correspondence with the Standard Model's content is stated explicitly rather than left implicit*: one generation there is fifteen Weyl fermions in five multiplets of $\mathfrak{su}(3)_c \times \mathfrak{su}(2)_L \times \mathrm{u}(1)_Y$—sixteen with a right-handed neutrino—and the quark/lepton distinction is which of them carries the colour $\mathbf{3}$. *On this construction that distinction lands as a grading rather than as a count*[P14R20]: the triality class is what separates the two, a coloured constituent carrying a non-zero class and a lepton the zero one, and no counting argument in this sector distinguishes them.

This sector fixes the number of such families, their chirality and the S_3 relating them, and fixes nothing about their internal content: not the multiplet count, and not the colour or isospin assignments.

Two items it does bear on: the wall structure requires each generation's content to be anomaly-free on its own—there being no bulk gauge field for inflow to come from—and the observed content meets that condition on every count and only as a complete set; and given the colour and isospin assignments, that same condition fixes the hypercharges uniquely up to normalisation—*so hypercharge is not independently undelivered, but it is not independently delivered either, and the distinction is the theorem above*: the assignments it is given are the chiral $2 + 1 + 1$, which is exactly what the no-go excludes on the achiral member this sector builds. On that member the structure is vector-like, the anomaly conditions are automatic, and they fix no hypercharge at all.

Hypercharge therefore rests on the same step as the fifth multiplet—and that step has been taken, in the negative. The chiral member *permits* the asymmetric action and does not *select* it[P14R53], so the $2{+}1{+}1$ the anomaly solution is handed is not delivered on either member. *What that settles is where the hypercharges come from*: the computation is a correct anomaly solution, and its premises—the multiplet structure and the existence of the Yukawa couplings—are the Higgs sector's. This sector contributes the requirement that each generation be anomaly-free *on its own*, there being no bulk field for inflow to come from; the other contributes what is being made anomaly-free. *The hypercharges are therefore not independently undelivered and not independently delivered: they are the composition of the two, and this construction's half of it is the constraint, not the content.*

And the two vector-like results should be read together, because they are independent and this sector escapes only one of them. The boundary paper's is the Atiyah–Hirzebruch index obstruction, whose load-bearing hypotheses are compactness and a *connected* gauge isometry, and whose one opening is the disconnected orientation parity the theorem cannot reach—the opening this sector is built in [6]. The obstruction above uses neither compactness nor connectedness: it needs only that the exchanging involution be a realised symmetry commuting with the species grading. *So building in the disconnected component clears the first and not the second*, and the polarised cut is caught by the second alone. The chiral member is what clears it, the conserved twist being the invariant that separates them. *The zero-mode is one Weyl mode per wall, not fifteen; the index counts families, not the states within them.*

Carried across the branch point at $r = 0$ the structure is inherited rather than remade: the reassignment fixes the expansion rate and does not touch the leaf-carried content, so the generations cross unchanged; the family *originates* at the crest, where two of the three roots merge and the S_3-symmetric locus splits undercritically into three; and the crest is a fixed point of S_3 but *not* of R, so the parity is untouched, the inherited matter stays chiral, and no matter/antimatter asymmetry is a cosmogenesis event—it stays the ordinary route. *Fourth, and reached after the three above, the discrete opening delivers more than a flavour skeleton.* The bundle the operator acts on is not a bundle of the substrate: every ambient candidate is real, and a real bundle's complexification carries a parallel conjugation, so its holonomy lands in the real form and none can carry $\mathfrak{su}(3)$. The module is the *branching* itself.

Because the wall is a wall *of a hinge*, the construction carries three signed areal radii rather than one, so a wall crossing branches only the vantage that owns it and the monodromy is diagonal in the vantage basis and non-abelian across it; the smallest connected group containing the three wall monodromies and the hinge 3-cycle is $SU(3)$, with the lap as its centre. Second quantisation—the ordinary step, available because the three wall modes are one operator's kernel and therefore identical particles—places a three-fermion state in the exterior cube, one-dimensional and generated by ε, and returns baryon 1, diquark 0, meson 1[L18]: every channel the Standard Model has, with the configuration group *selected* rather than chosen.

And the negative half is as sharp and is stated with it: the bundle is *flat*. Flat holonomy supplies exact selection rules and no curvature, so the construction delivers the discrete content of colour and supplies no force—the geometry quantises and does not couple.

Two further facts about the sector's discrete group are recorded. It is larger than $D_6 = S_3 \times \mathbb{Z}_2$: the residue pairing the horizon roots' surface gravities carry has a holonomy about the Nariai points which, adjoined to the walls' S_3, closes the Weyl group of the substrate's own complexified isometry algebra—so the excess is substrate-derived rather than assumed. *The derivation is sound and the surprise sits elsewhere than it first appears*: a group of order twenty-four with S_3 image and Klein four-group kernel is what *any* cubic's residue pairing returns, so the closure is forced by the cubic and not by this one[7L3]. *What is not forced is that the substrate's Weyl group is that same group*, and that is a rank-three coincidence: $|W(D_n)|$ runs $4, 24, 192, 1920$ and meets 24 only at D_3, where $\mathfrak{so}(6,\mathbb{C}) \cong \mathfrak{sl}(4,\mathbb{C})$. *The match holds where it holds and fails immediately on either side*—at $\mathfrak{so}(8,\mathbb{C})$ the cubic still gives 24 and the Weyl group gives 192. And that holonomy acts on the walls' chiralities, which are the σ_y eigenvalues of Proposition 1, changing their signs only in *pairs*: the observed configuration is therefore the unique member of its orbit with all three walls at one chirality, and $\dim\ker_-$ cannot be reached from $\dim\ker_+ = 3$ by any number of loops, three and zero lying in different parity classes. The generation count is thus not merely what the index returns but what no holonomy of this connection can move.

The count is forced within CR by the programme's own criterion of necessity, not by an added axiom, and the honest edge is stated plainly rather than gestured at: a framework declining that criterion reads the natural single-hinge index—one—not three. A further consequence is drawn in Section 7, and it runs the other way to the rest of the paper: because the fold the count reads is $D-1$ in a D-dimensional cut, and because the horizon relation collapses to a single multiple-angle only at four and five dimensions while the mass-parity that grades chirality exists only in even dimension, four is the only dimension carrying both a generation count and a handedness. Within CR, and at the same altitude as the rest, three generations and four-dimensional spacetime are one fact read at two ends.

1 The question, and what is and is not claimed

One statement in this paper does not fit that description and is flagged here so it is not read as more than it is. Section 7 draws a consequence about the *dimension* of the cut—four dimensions being the only one in which the sector's own two deliverables, the count and the chirality, both exist. It is at the same altitude as everything else here, forced within CR and owing its correspondence to the world; it is not a demonstration that the world is four-dimensional, and it carries one assumption of its own, that the metric function in a general dimension is the standard Tangherlini–de Sitter form.

The slicing operator of CR generates the spherically symmetric, stationary sector of general relativity as the perspectival reading of a single maximally symmetric substrate [10, 11, 5]. The boundary paper [6] asks whether the *matter* content can be read off the same substrate, and returns a precise negative for the gauge sector: $\mathfrak{su}(3) \not\subset \mathfrak{so}(5,1)$, structurally, so the Standard Model gauge group is not a continuous substrate isometry. What that boundary leaves standing is the substrate's *discrete* structure—the orientation parity and the threefold symmetry of the slicing—recorded there as the one opening through which matter might enter, and carried as a conjecture.

This paper takes up that opening concretely. We put a Dirac field on the slicing curve and compute what its zero-mode content is. The result is a fermion sector with exactly the discrete shape the Standard Model wears: three chiral generations carrying a family symmetry. We are careful throughout to separate what is delivered from what is not:

- **Delivered.** The generation *count* (three), the *chirality* of each generation (γ^5), and the S_3 relating the three walls—a *within-state* index rather than a family symmetry, the generations' own threeness being the turnaround's deck $\mathbb{Z}_3$ (Section 6.1), with the wall structure fixing the *number* at either seat. These are the discrete flavour structure [21]. *And, reached after them, the discrete content of* ***colour***: the module the operator's colour structure acts on is the branching rather than any bundle of the substrate; the three wall monodromies with the hinge 3-cycle generate $SU(3)$; and second quantisation on the wall kernel returns baryon 1, diquark 0, meson 1, the configuration group selected rather than chosen. With them, the *kind* of each—why the chirality descends gauged and the family global—which Section 5 reads off the two relations the substrate's waist admits.

- **Not delivered.** The *coupling*: the bundle above is flat, so the construction supplies colour's exact selection rules and no force—it quantises and does not couple. Weak isospin as a *gauging*: T is a discrete horn swap and delivers a species label, not $SU(2)_L$'s chiral action, the two occupations differing on the right-handed pair (Section 6.1). Hypercharge's normalisation is fixed by the winding rather than by the anomalies, which are homogeneous. And the mass spectrum: the zero-modes are massless, and their splitting is electroweak physics, external to the geometry.

The generation count is *forced within CR*, in the precise sense made explicit in Section 3: it follows from the maximal-symmetry principle that defines the programme, applied to the matter construction as the same principle is already applied to the substrate and to the discrete breaking [5]. It is not a theorem a framework lacking that principle must accept. That qualification is the honest content of the word "forced" here, and we do not inflate it.

2 The chiral wall

On the slicing curve the areal radius r is a *signed* coordinate: the curve runs forward through the seam and backward through $r = 0$ as one closed object—the *cosmogenetic bead* the framework proves one closed curve [11], on which this sector's chiral wall is built—$r = 0$ being a branch point the field crosses smoothly, not a barrier [7, 13]. The massless radial Dirac problem separates into a first-order pair with superpotential

$$W(r) = \frac{\lambda\sqrt{f}}{r}, \qquad f = 1 - \frac{2M}{r} - \frac{r^2}{\alpha^2}, \qquad \lambda = j + \tfrac{1}{2}. \tag{1}$$

This superpotential is the explicit output of the leaf frame: the orthonormal tetrad $e^0 = \sqrt{f}\,dt$, $e^1 = dr/\sqrt{f}$, $e^2 = r\,d\theta$, $e^3 = r\sin\theta\,d\varphi$ of the $M \neq 0$ slicing geometry has radial–angular spin connection ${\omega^2}_1 = {\omega^3}_1 = (\sqrt{f}/r)\,e$, and the radial first-order pair carries $W = \lambda\sqrt{f}/r$ *in the tortoise* *variable* $dx = dr/f$ (receipt: P14R2); read instead against the frame derivative $\sqrt{f}\,d/dr$ the same operator carries $W = \lambda/r$, the two being one operator written on two derivatives and giving the one logarithmic derivative $\psi'/\psi = \lambda/(r\sqrt{f})$[7]. *The wall's structure does not turn on which is written*: $\sqrt{f}/r$ vanishes at the horizons ($f = 0$), and both forms are odd under the reflection that carries $(r, M) \mapsto (-r, -M)$—so the sign change at the throat, the count and the chirality below rest on either. *What does turn on it is the near-throat exponent*, which is taken up where the index is read. Because r is signed, W changes sign at $r = 0$: a domain wall [18]. *A superpotential of this kind invites the question whether the pair is* shape invariant — *the condition under which the supersymmetric ladder generates the whole spectrum — and it is not.* Writing $W = \lambda\sqrt{f}/r$ and $V_\pm = W^2 \pm \mathrm{d}W/\mathrm{d}x$, the requirement $V_+(\lambda) = V_-(\mu) + R$ with R independent of r admits no constant shift $\mu(\lambda)$ except the degenerate $\mu = -\lambda$, which returns $V_+(\lambda) = V_-(-\lambda)$ exactly and so $R = 0$: the partners coincide and there is no energy shift to climb. *So the exactness available here is the zero mode's and not the spectrum's*—the mode below is obtained because W changes sign, which is topological, and not because the problem is solvable, which it is not[L14]. That is why the count that follows is read from an index rather than from a list of levels. The zero-mode equation $H\psi = 0$ for $H = -i\sigma_x\partial_x + m(x)\sigma_z$, with $m(x)$ the wall profile crossing zero at the throat, integrates to

$$\psi' = -m\,\sigma_y\,\psi \quad \Longrightarrow \quad \psi(x) = \exp\Big(-\int_0^x m\,dx'\Big)\,\chi_+, \tag{2}$$

with χ_+ the $\sigma_y = +1$ eigenspinor. For a wall $m = \tanh(x/a)$ this is $\psi = \cosh^{-a}(x/a)\,\chi_+$, an exact normalizable solution; the conjugate branch χ_- grows as $\cosh^{+a}$ and is rejected [19].

The norm in which the mode is normalizable is the one CR's ontology selects, and the point is load-bearing. The fermion is a field on the evolving spatial leaf—the existent of the layered ontology [11], of which the spacetime is a projection—so its norm is the induced proper-distance measure on the cut, $\int|\psi|^2\,d\ell$ with $d\ell = dr/\sqrt{|f|}$.

In that measure the turning points ($f = 0$, the horizons) lie at *finite* proper distance and the $r = 0$ crossing is an integrable $\sqrt{\ }$-singularity, so the bound state is genuinely normalizable, as the tanh model captures.

In the conserved *spacetime* Dirac norm (tortoise measure $\int|\psi|^2\,dr_*$, $dr_* = dr/f$), by contrast, the same static mode is *not* normalizable: the horizons sit at infinite tortoise distance, where the mode tends to a constant.

The two are not interchangeable, and CR reads the fermion as a mode of the existent leaf, not a propagating spacetime field carrying the tortoise norm—on the leaf it is a bound state. *The direction of that failure is fixed rather than symmetric, and saying so sharpens the caveat rather than softening it.* On the static region between the horizons $\mathrm{d}\ell/\mathrm{d}r_* = \sqrt{f}$ is bounded, so $L^2(\mathrm{d}r_*) \subset L^2(\mathrm{d}\ell)$: the leaf norm is strictly the weaker condition, and the selection it performs is done *on that static region itself*—where the leaf-measure amplitude runs as $r^{\mp\lambda}$, so that one chirality decays and is bound while its conjugate grows and is rejected P14R3—and not at the horizons, where the two norms disagree about every bounded mode alike.

Nor is it done at $r = 0$: on the operator the frame actually gives, both chiralities approach the throat as a bounded phase rather than parting company there, the crossing being an integrable square-root singularity that selects nothing[7].

The count is unaffected—one bound chiral mode per wall, as Proposition 1 states—and what changes is only where the selection is located. *And the static mode's failure in the tortoise norm is a statement about that mode rather than about the sector*: a modulus tending to a constant at infinite tortoise distance is the plane-wave asymptotic, so it is the normalisation condition of a continuum state and not an obstruction to one.

The $\omega \neq 0$ problem the same superpotential defines is accordingly an ordinary short-range scattering problem—W and both partner potentials $W^2 \pm \mathrm{d}W/\mathrm{d}r_*$ decaying exponentially in r_* at the surface gravity, by the same simple-root exponential that carries a thermal spectrum, with unitary transmission across the tower[7]. *What that supplies is the radial continuum and not the sector*: the quantised field, its mode completeness, and the join between the static region's continuum and the wall—which sit in different regions—remain the undertaking the corpus names [5, 6]. The geometric half of that join is not itself open: the wall's locus is the signed radius's own zero, and the slicing construction carries a C^∞ continuation across it—"a branch point and not a barrier" [7, 13]—so what remains is the matching of modes across a locus the geometry already crosses, and not the crossing. Direct solution of the exact zero-mode across a full undercritical slicing confirms both limits (divergence at the horizons in the tortoise measure, convergence in the leaf's proper measure). Hence:

Proposition 1. *Each throat wall binds exactly one normalizable chiral zero-mode. Its chirality is a definite σ_y eigenvalue, whose sign is the sign of the signed-radius flip; that operator is the diagram-automorphism parity $R = \gamma^5$ [9].*

This is the $\mathbb{Z}_2$ (chirality) half of the substrate's discrete structure, realised as a bound state, not posited. *One feature of W deserves comment, since $\lambda = j + \frac{1}{2}$ appears in it and might be thought to carry multiplicity.* Written on either of the two paired forms above the first-order pair gives one logarithmic derivative, $\psi'/\psi = \lambda/(r\sqrt{f})$, so the zero mode is $\psi = \exp\!\big(\lambda \int \mathrm{d}r/(r\sqrt{f})\big)$ rather than the tanh model's exponential. *On the static region between the horizons that integral is finite and the amplitude runs as a power of* $|r|$, which is where the branch selection is made; *near the throat it is not a power at all,* $f \to -2M/r$ making the exponent $\propto i\sqrt{|r|}$ and the mode a bounded phase[7].

That the mode is a power of the signed radius, and that the signed radius is a cube root at the wall, together decide a question about the mode that is worth settling here because it is not obvious and its answer sorts the spectrum.[P14R25] Near the wall $f \to -2M/r$ gives $r^3 = \frac{9}{2}M\ell^2$ in the leaf's proper measure, so a crossing of the wall is a half-loop in ℓ and sends $r \mapsto \omega r$ exactly, $\omega = e^{2\pi i/3}$. *A mode transported across a wall therefore returns multiplied by $\omega^{-\lambda}$.*

The consequence is a dichotomy rather than a uniform statement: a mode with $\lambda \equiv 0 \pmod 3$ is unaffected and is an ordinary single-valued function of position, while a mode with $\lambda \not\equiv 0$ is *not*—its value depends on which path reached the point, two paths differing by one crossing disagreeing by $\omega^{-\lambda}$. *So the spectrum divides into modes that are functions of where they are and modes that are functions of how they got there*, and the divide is exactly the residue λ mod 3 that Section 3.1 identifies as the quark/lepton discriminant. A further consequence follows without extra input and is stated at the same weight: since a field on the manifold must be single-valued, a configuration whose total λ is not divisible by three is not a field on it at all, so only combinations of vanishing total residue exist as configurations. *We record the shape of that statement and claim nothing from it*: it is the shape of a confinement condition, and whether the residue it grades is the physical colour is the question the boundary paper walls and this paper does not reopen [6]. *And the shape is tested against the observed spectrum rather than left as a shape.*

Taking the condition at face value—a configuration exists only if its total residue vanishes, and confinement is the failure of a winding to close its lap—and enumerating the low-lying combinations returns agreement on eleven of eleven: meson, baryon, antibaryon, tetraquark, pentaquark and dibaryon close and are observed; a free quark, a diquark, $q\bar{q}$–$\bar{q}$, $qqqq$ and qqq–$\bar{q}$ do not close and are not. The thirds themselves follow from three geometric constraints with no Standard-Model input. We still claim nothing about whether the residue is physical colour—that wall stands—but the condition it defines is *a test the spectrum passes and not only a shape.* Near the branch point the leaf measure behaves as $\mathrm{d}\ell = \mathrm{d}r/\sqrt{|f|} \sim \sqrt{|r|/2M}\,\mathrm{d}r$, an integrable square root, and the mode is a bounded phase there, so *the throat admits both branches and selects neither. The selection is made on the static region*, where the exponent $\lambda\int \mathrm{d}r/(r\sqrt{f})$ is real and finite: one branch decays and is bound, its conjugate grows and is rejected[P14R3]. At the horizons the measure carries an integrable inverse square root and ψ is finite, so the norm converges there as well[P14R13].

Hence exactly one branch is bound for each j, and the tower over j is infinite. The angular spectrum is worth stating as the honest negative it is[P14R48]: λ fixes a *grading* and not a *content*—it indexes partial waves of one field and supplies no multiplet structure, so nothing in the angular sector *has yet been* read as internal content—and the receipt cited is explicit that this forecloses nothing: a graded infinite tower whose lowest level reproduces one generation remains a live reading, and it is closed, if at all, by the Kaluza–Klein negative below rather than by the spectrum itself. That is the ordinary angular decomposition of a four-dimensional field, not additional multiplet content: j labels the partial wave of each generation's mode, and the index of Section 3 counts walls, not partial waves. *We record this because the appearance of λ in the superpotential invites the opposite reading*— that the wall problem might supply the internal structure of a generation—and it does not. *And it is not a Kaluza–Klein tower either, which is the other reading the shape invites.*[P14R47] A Kaluza–Klein tower requires an *internal* space to reduce on; λ indexes the Dirac operator on the transverse S^2, and that S^2 is the $r^2\mathrm{d}\Omega^2$ of the four-metric—it is spacetime, and $2 + 2 = 4$ exhausts the cut, so there is nothing left to reduce on and the levels are not distinct four-dimensional fields.

The one genuine extra direction, the cut-normal, cannot supply a tower either, and for the opposite reason: it is a single *non-compact* dimension, giving a continuum rather than a discrete tower. The internal content remains where Section 3.1 places it, on the far side of the gauge wall. *That undelivered content is where the programme's cosmological side arrives too, and the two arrivals are one.* The cosmological companion inherits two composition data across the branch point: a ratio of energy densities taken over the matter as a whole, and a ratio of numbers taken over the baryons alone [1, 14]. *Passing between them requires knowing which fermions the handover delivers, in what numbers and at what masses*—which is this section's undelivered content, approached from the cosmological side. That the two data appear there as independent empirical inputs is therefore not a separate gap: *it is how the missing representation content presents itself to a cosmology.* We record the identification and claim no derivation from it; what it fixes is that a route across the wall would settle both, and that neither is settled without it. *It is worth stating plainly where the gap sits, since the index of Section 3 is easily read as producing more than it does.* That index counts zero modes of a Dirac operator acting on a bundle, and it returns one mode per wall *per internal state of that bundle.* The count of three is a count of walls; the multiplicity of a generation is the bundle's rank. *So the question the undelivered content amounts to is a question about the bundle*: what does the operator act on, and what fixes it? On the ordinary route the bundle is imposed—one posits the gauge bundle and computes. On a geometric route the substrate would supply it, and the substrate is not short of bundles: the tangent bundle of the five-dimensional embedding space, the spinor bundles it carries, the normal bundle of the wall itself, and the two ruling bundles the framework's construction already draws [11].

That list, however, can be settled at a stroke, and settling it moves the question rather than answering it.[P14R26] The obstruction is one nobody had written down: $\mathfrak{su}(3)$ requires a *complex* rank-three module, and a *real* bundle's complexification carries a parallel conjugation, so its holonomy commutes with that conjugation and lands in the real form—$\mathfrak{so}(3)$, of dimension three, with no room for eight. Every member of the list falls to that or to rank. The embedding tangent bundle is real of signature $(5, 1)$, and a metric-compatible complex structure J splits a space into η-orthogonal definite planes $\mathrm{span}\{x, Jx\}$ and so forces both signature blocks to be *even*, which 5 and 1 are not; the spinor bundle has rank four; the rulings are lines. The wall's normal bundle alone has the right rank, and it is reality rather than any decomposition that removes it—*the* 2+1 *splitting one is tempted to use is covariant only under the stabiliser of a chosen cut direction, whereas under the full transverse* $SO(3)$ *that bundle is the vector representation and is irreducible. What the column shows is that every candidate was built from the substrate's ambient geometry, and ambient geometry is real: the question was never which real bundle but where the complex structure is.* It is at the branch point.

And finding it there requires taking this paper's own definition of the wall more literally than the winding computation had been taking it.[P14R28] The wall is where the signed areal radius vanishes, and that radius is fixed *relative to the hinge about which the slicing plane swings*—so there are three of them, one per vantage, and three branch loci rather than one. The companion slicing paper states as much at the outset: the matter construction places a plane on each of the three hinges, giving "three throat walls at distinct points of the throat circle *with disjoint support*" [7].[P14R44] *The distinction had been invisible because the two readings agree on the label and differ only on the group*: with one radius the cover is cyclic and its monodromy abelian, which is why the search above returned only a $\mathbb{Z}_3$; with three, crossing a wall branches only the vantage that owns it. *That the corpus appeared to hold both is not a conflict between the papers but two constructions*—for the vacuum family the three hinges are equivalent vantages and a single swinging door charts everything, so a single radius is the right bookkeeping there; for the matter sector the planes are placed rather than swung. Independently, eliminating $-X_0^2 + X_1^2$ between static de Sitter's $\alpha^2 - r^2$ and the equatorial section's $-X_0^2 + X_1^2 + X_2^2 = \alpha^2$ gives $r^2 = X_2^2$, so on the throat circle $r_j = \alpha \sin(\varphi - \theta_j)$, returning this paper's own "the back, $X_1 = -\alpha$" rather than being fitted to it, and vanishing at each vantage's own wall and neither other.

Nothing then has to be adjoined. A single wall monodromy has determinant ω, so the unimodular content sits in the ratios, and $\gamma_A \gamma_B^{-1}$ is a non-central diagonal of determinant one; it fails to commute with the three-cycle permuting the vantages, the pair's commutant is the scalars, and the pair preserves no symmetric bilinear form—so the smallest connected group containing the three wall monodromies together with the hinge three-cycle is $SU(3)$ itself. *The two generators are this paper's two threes doing different jobs, needed together for the first time rather than kept apart: the diagonal one is the three walls, the permuting one is the three hinges, and neither alone is more than* $\mathbb{Z}_3$ *or* S_3. A full lap, being the product of all three, comes out as $\omega^{-\lambda}$ times the identity—*the centre*—which is the sense in which the single-valuedness criterion of §2 is a statement about a lone constituent and not only about a pair of routes. *It also settles what cannot work*: the three colours cannot come from the three sheets of a single cover, since a mode has one triality and would then have one colour.[P14R33] *Nor is this rank-three object the one two paragraphs above.* That one was rank three because a cube root has three branches and it graded the tower by triality; this one is rank three because there are three hinges. A single mode occupies one line of the first and exists at *all three* vantages of the second, so all three colours of one constituent carry the same triality—which is the Standard Model's own relation between the centre, carried identically by every state in a representation, and the index running within it.

The reading owes an account of the null legs, since it is on the legs and not the equatorial arcs that this section reads the crossings.[P14R29] The leg joining two hinges grazes the wall owned by the *third*—three legs, three walls, each leg opposite its wall—which is this section's own observation that a hinge's legs never graze its own wall, seen from the other side. The assignment reproduces a multiplicity it was not built from: two punctures per hinge give $2 \times 2 = 4$ legs per hinge pair, all sent to one wall, returning the $12 = 3 \times 4$ of the twelve null legs. So each leg meets exactly one wall, the three legs meet the three walls once each as one lap does, and the closed bound triple acts by the centre. *What looks at first like the reading's difficulty is its content*: a constituent at one hinge is labelled by the leg it is not on, and a label that cannot be read locally is what route-labelling means.

What the resulting selection rule reaches, and what it does not, has to be said in one breath.[P14R30] The centre gives $N - \bar{N} \equiv 0 \pmod 3$ and nothing finer—it does not distinguish a triple spread one per hinge from one doubled on a single hinge, exactly as the centre of $SU(3)$ does not—so triality zero is necessary for a colour singlet and not sufficient, and the one-per-hinge condition comes not from it but from the causal trichotomy of the six hinge-ends.[P3R49] *The two conditions arrive from unrelated places and neither does the other's work.* Sufficiency needs the invariants of the group just identified, and counting them on the tensor cube returns too many and, under the full holonomy group, the wrong one—*which is a feature of gradings rather than a defect of this construction*, since a grading records how many constituents occupy each slot and nothing about their order or sign, whereas ϵ_{abc} is a statement about antisymmetry.[P14R31] *Antisymmetry, however, is not a gauge datum, and this section has been carrying what supplies it since Proposition 1.*

The three wall modes are the kernel of one Dirac operator, so they are states of a single field and therefore identical particles—not by an added postulate but by what having a single operator means.[P14R32] A three-fermion state is accordingly not a general tensor; it lives in the exterior cube of the kernel, and since Λ^3 of a three-dimensional space is one-dimensional with the totally antisymmetric generator, *the surplus invariants are not states a symmetry fails to forbid—they do not exist*: the one with all three constituents on a single vantage vanishes by the Pauli principle, and the symmetric combination is not in Λ^3. Counted on the exterior powers the channels come out *baryon one, diquark none, meson one*—each exactly what $SU(3)$ gives, with the diquark a prediction rather than a fit, nothing here having asked that Λ^2 carry no invariant. *And the count decides a question this construction could not otherwise settle*: of the three defensible readings of which loops a bound configuration may traverse, exactly one returns all three channels correctly, and it is the unimodular group—the same group the $SU(3)$ entailment above reached by an unrelated argument. *One consequence bears on §2 in its favour*: under that group the baryon line has trivial holonomy and is invariant exactly, while the one-particle space carries no invariant at all and the action on it is irreducible, so a lone constituent does not return to itself up to a phase, which a ray could absorb, but returns *a different vector*. The single-valuedness criterion is thus doing its work without needing any distinction between fields and states to prop it up.

What remains is then not a defect but a description. All of this is holonomy—a flat bundle and exact selection rules—so the construction supplies the discrete content of colour and supplies no force, which is what the winding label was separately found not to supply.[P14R23] *And the flatness is a complete obstruction rather than a stage not yet reached, which is worth showing rather than asserting, since a reader is otherwise entitled to hear it as a promise.*[P14R51]

The holonomy group generated by the three wall monodromies together with the hinge three-cycle is *finite*, of order 81—necessarily so, since the holonomy is branching and a branch structure has finitely many sheets. *The flatness is then not a stipulation but a consequence of that finiteness*: by Ambrose–Singer the holonomy algebra is spanned by the curvature, so a finite—hence zero-dimensional—holonomy group forces $F = 0$ identically, which is the same theorem the algebroid paper invokes for the reverse reading [9][L3]. And a finite group in characteristic zero has vanishing first cohomology with any coefficients, so the representation does not deform as a representation of its own holonomy.

Deformations of the fundamental group's representation that keep each wall's conjugacy class fixed are counted directly and the moduli space is *zero-dimensional*: a wall branches one vantage and leaves the other two alone, so its monodromy carries a repeated eigenvalue and its class is subregular, of dimension four rather than the regular six; three regular classes on a three-punctured base would have left a two-parameter family, and the disjointness of the vantages' supports is exactly what removes it.

But the dimension is a bonus and not the argument. The moduli space of flat connections consists of flat connections, so a deformation within it changes which flat bundle one has and not whether there is a field strength; obtaining curvature means leaving the flat locus, and no holonomy datum can take one off it, because holonomy is precisely the complete invariant a flat connection has. *Leaving it requires a variational principle, and a Yang–Mills term in four dimensions carries a dimensionless coupling that a single length cannot build*—the substrate's one invariant being $\alpha = \sqrt{3/\Lambda}$, whose energy $\hbar c/\alpha$ sits some forty-one decades below the strong scale, and in the infrared rather than the ultraviolet direction from it.

So the position is not that the coupling is unbuilt but that a coupling is not the kind of thing a holonomy supplies, which is the same verdict the winding received one level up and for the same reason. *What is not excluded here is a mechanism that is neither holonomy nor isometry; the isometry route is excluded separately, and the honest statement is that no third mechanism has been named, and naming one remains open. What can be said about one before it is named is more than nothing, and it is the dimensional sentence just used, which mentions no route.*

That sentence constrains the *target* rather than the mechanism: whatever produces the connection, what it must end in is a four-dimensional Yang–Mills term, and that term requires a dimensionless number the substrate's ledger does not carry—the one physical length being α and not ℓ_P, whose ratio $\alpha/\ell_P \sim 10^{61}$ is a number in gauge-units and not a tuning [5]. *So what a third mechanism must deliver is therefore a fixed pure number rather than a free parameter*, and a candidate is accordingly falsifiable against one quantity rather than searched for in an unbounded space[7].

And the wall is four-dimensional and nothing else: dimensional consistency of $\int \mathrm{d}^D x\, F^2/g^2$ gives $[g^2] = L^{D-4}$, so at the substrate's own $D = 5$ a Yang–Mills coupling *is* a length and the substrate has exactly one—the obstruction appearing only after the descent, which locates where such a mechanism would have to act without asserting that anything stands there. *The same counting is what distinguishes this from a general ban on geometry producing fields*: the Einstein–Hilbert coupling has $[1/16\pi G] = L^{2-D}$, dimensionful in every dimension, so a single length builds it and gravity is precisely the case the argument does not touch. *One objection to a winding label deserves its answer here, since it is the natural one*: a route-dependent quantity is not readable at a place, and electric charge is.[P14R24] The winding splits as $w = n + \phi$ with n an integer lap count and ϕ a $\mathbb{Z}_3$ class, and the two pieces transport differently—one wall crossing multiplies the field by ω^{triality} while a full lap is three crossings and $\omega^3 = 1$—so the *integer* part is route-independent and the fractional part *is* the monodromy.

Hence w is locally readable exactly when it is an integer, exactly when the triality vanishes: a fractionally charged object is precisely one whose charge is route-dependent, hence not readable, hence not free.

The $\mathbb{Z}_3$ is therefore the obstruction to charge being observable rather than a rival to it. *One qualification, entered here because the receipt-vs-sentence audit found this paper claiming otherwise*: the agreement between "is a field" and "has integer charge" is not an independent check that could have failed.

On the identification used here the triality class *is* the fractional part of the charge, so the two predicates are the same predicate and agree on any rational whatever. *What the argument establishes is the implication, not a test of it*: an object with non-zero triality is route-dependent, hence unobservable, and colour confinement is that statement read as a selection rule. A genuine test computes the triality from the colour content independently of the charge, and that test is available here: the wall monodromies generate the centre, and triality *is* the centre's action, $\rho(\omega I) = \omega^t I$ with $t = (p-q) \bmod 3$ on a tensor of p upper and q lower colour indices. Evaluating it on the channels above returns $t = 0$ for the meson and the baryon and $t = 2$ for the diquark—which is the configuration count Λ^3 returns, obtained a second way and with no charge on either side[7].

What the two routes agree on is the *selection*; the identification of the triality class with the charge's fractional part remains the assumption it was, and is not what this test establishes. *The winding itself, its closure rule and the derivation of the thirds are the companion slicing paper's* [7]: there the graze points cut the throat into three equal arcs, the two routes between punctures at different hinges differ by exactly one lap, and imposing closure on both horn-headed arrangements forces the winding into thirds with no Standard Model input. *One question that construction leaves is properly this paper's, because it is about matter content rather than about the lap*, and it has a clean answer with a clean negative before it.[P14R22]

What exchanges *uud* and *udd* is not charge conjugation: $w \mapsto -w$ carries the quark class to the *antiquark* class, and no lap shift repairs it, because conjugation reflects through the origin while the (u, d) pair is not centred there.

Solving for the involution that does it—rather than positing one—returns a unique *affine* map, $w \mapsto \frac{1}{3} - w$, whose constant is fixed by the pair itself. *And it is the Standard Model's own map*: with $Q = I_3 + Y/2$ the isospin Weyl reflection at fixed hypercharge is $Q \mapsto Y - Q$, and the constant came out equal to the quark doublet's hypercharge $\frac{1}{3}$ with nothing fitted. Its two factors are both already in hand—$w \mapsto -w$ is the slicing paper's independent charge $\mathbb{Z}_2$, and $w \mapsto w + \frac{1}{3}$ is one graze-point crossing, which is one wall. *And the exchange sits inside the configuration's own group, so nothing needs adjoining*: it is the horn swap T, a substrate isometry belonging to the order-twelve group already in hand. *The offset is no obstacle to that*, the apparent difficulty—that $\mathrm{Aut}(A_2)$ acts linearly while a map with a non-zero constant does not—arising only if the two are compared in a single space, which they do not share. *Two things are worth marking about the standing of the whole*: that the vantage rotates the areal direction is a reading of the sentence quoted above rather than a computation of it, though the companion paper's disjoint-support statement carries the same content independently; and the exhibit of the three radii is at the $M = 0$ member, where the cut is the substrate.

We have not examined whether any of these has the rank and the structure the Standard Model's content requires beyond what the preceding paragraph settles, and we do not claim that the question is closed. One further negative result bears on it and is recorded above: the angular decomposition does not supply multiplicity, since $\lambda = j + \frac{1}{2}$ labels partial waves and each contributes exactly one bound mode. *That result is stronger than a fact about the sphere we wrote down, because no other transverse space is available to the construction.*[P3R39]

Run with the transverse curvature k left free, the operator of the companion slicing-operator paper returns as its entire kernel $f = k - 2M/r - \Lambda r^2/3$, so the operator alone does not fix k; but k enters as an additive constant independent of M, and the family passes through $2M = 0$ six times per swing, at which marks the cut *is* the substrate, $f = 1 - r^2/\alpha^2$, whose constant term is unity—so $k = +1$ is an output of the construction rather than a choice of chart. Constant curvature $+1$ leaves the global form S^2/Γ, and there Γ is fixed by the spin structure the Dirac operator requires: every non-identity element of SO(3) is a rotation about an axis meeting the sphere in two fixed points, so no non-trivial subgroup acts freely, and the one free finite action—the antipodal map, which is not in SO(3)—has the non-orientable quotient $\mathbb{RP}^2$, which carries no spin structure at all.

The transverse space is therefore the round S^2 and nothing else, and the tower over λ is infinite for that reason rather than by stipulation. Nor would any alternative have helped: an orbifold quotient $S^2/\mathbb{Z}_n$ thins each level's degeneracy from 2λ to roughly $2\lambda/n$ and removes no level, a quotient leaving untouched the local geometry that sets λ. *So the source of the tower's multiplicity is closed by exhaustion and not by inspection*—a definite question about a definite object, rather than an absence. *Two questions have been called "the bundle question" and only one of them is this one*: what module the operator's colour structure acts on is settled above, and it is the branching rather than any bundle of the substrate; what supplies the *multiplicity* of the tower is the question this paragraph leaves open, and it is not a verdict on the sector. *One of the substrate's bundles can be checked immediately, and doing so also sharpens what is being asked.* The operator above acts on the leaf's four-dimensional spinor bundle. The substrate's own spinor bundle is five-dimensional in base, and in odd dimension the Dirac representation is irreducible of dimension $2^{\lfloor 5/2 \rfloor} = 4$, restricting to a codimension-one slice as the four-dimensional Dirac representation—*the same rank the operator already uses*[P14R11]. So the substrate's spinor bundle supplies no multiplicity beyond what is in hand. The remaining natural candidates are of low rank as well: the tangent bundle of the embedding space has rank five, the wall's normal bundle rank one, and each ruling family rank one.

But the check also corrects the question, and the correction matters more than the result. Fifteen is a sum of representations, not a bundle rank: a rank-fifteen bundle carrying no $\mathfrak{su}(3) \times \mathfrak{su}(2) \times \mathrm{u}(1)$ action would supply nothing, while a rank-three bundle carrying colour would supply the essential thing. *So the question is not which bundle has the right rank but what would furnish the group action*, and the low ranks recorded above are therefore not evidence against those bundles—they are simply not the relevant measure. *We record one candidate examined and the question restated*; the remaining candidates, and the possibility of structure arising by means none of them carries alone, are untouched here.

2.1 A condition the construction imposes on its own content

One constraint follows from the construction as it stands, and it has not been drawn. The wall of Section 2 localizes chiral zero modes on a codimension-one surface of a higher-dimensional bulk, which is the configuration of Callan and Harvey [19]—cited above for the rejection of the growing branch, but carrying a second consequence. Chiral fermions confined to such a wall have an anomaly, and consistency requires it be cancelled either by inflow from the bulk or within the wall's own content.

Here there is no inflow to be had. Inflow proceeds through a bulk term built from a gauge field, and this bulk has none: the substrate is maximally symmetric vacuum geometry, and the gauge group is established not to arise as an isometry of it [6]. *So the bulk contributes nothing, and the wall's content must be anomaly-free on its own.* And because the walls are three separate loci carrying separate zero modes, the condition applies *to each wall*—which is to say *generation by generation*, not to the three together.

That is a requirement this construction imposes, and the observed content meets it non-trivially. A Standard Model generation cancels every condition—$[\mathfrak{su}(3)]^3$, $[\mathfrak{su}(2)]^2\mathrm{u}(1)$, $[\mathfrak{su}(3)]^2\mathrm{u}(1)$, $[\mathrm{u}(1)]^3$, and the mixed u(1)–gravitational—and cancels them only as a complete set: removing e^c leaves $[\mathrm{u}(1)]^3 = -1$, removing u^c leaves $8/9$, removing L leaves $1/4$[P14R18]. *And the Standard Model's anomalies cancel generation by generation, which is the pattern the three-wall structure requires and not the weaker one it might have required.*

And the condition's reach has a definite edge, which is worth drawing because the sixteenth Weyl fermion sits exactly on it. The right-handed neutrino $\nu^c : (\mathbf{1}, \mathbf{1})_0$ carries no colour, no isospin and no hypercharge, so its contribution to every one of the conditions above is identically zero: *fifteen is anomaly-free and sixteen is anomaly-free, and the condition returns the same number in both cases*[P14R19]. *So the one constraint this construction places on content cannot distinguish them*—and the generation count, being a graded index of wall-localised zero modes, counts generations rather than the content within one and does not reach the question either. *Both of this sector's handles on matter content are therefore blind to that state*, and a preference between fifteen and sixteen must be sourced from outside the construction, which is the representation-content step this paper declines.

We are careful about what this is. It is not a derivation of the content: anomaly freedom is a condition, and many contents satisfy it. *It is a condition the geometry supplies rather than one imposed to make a model work*, and it is the first of the undelivered items on which this sector says anything at all. It also has a definite falsifier: were a bulk gauge field to be found in the substrate, the inflow would be non-zero and the per-wall condition would relax, so the argument stands or falls with the absence established in [6].

The condition has a further consequence, which reduces the count of what this sector leaves undelivered. Section 3.1 lists three items—colour, weak isospin, and hypercharge—and treats them as three. *They are not independent.* Given the gauge group and the multiplet structure, the per-wall anomaly conditions together with the existence of the Yukawa couplings that give the fermions mass—a representation-theoretic requirement, on which assignments admit a gauge-invariant $\bar{\psi}\phi\psi$ term *at all* and not on any coupling's value—determine the hypercharges completely, up to a single overall normalisation. *That last qualification is not a formality, and the sector supplies what removes it.*[P14R46] Solved at general N_c the anomaly conditions are *homogeneous*—every hypercharge is proportional to one free q, and the cubic $[\mathrm{u}(1)]^3$ condition, not used in the solve, is identically satisfied for every N_c and so selects nothing—which means the anomalies fix ratios and can produce no scale at all. The winding closure is not homogeneous: its unit is one full lap, a closed circuit rather than a chosen normalisation, so it returns absolute values. The two agree where they overlap, the winding's $u = +\frac{2}{3}$ and $d = -\frac{1}{3}$ giving $q = \frac{1}{6}$ and the anomaly solution then reproducing all five Standard Model hypercharges exactly. Solving $[\mathfrak{su}(3)]^2\mathrm{u}(1)$, $[\mathfrak{su}(2)]^2\mathrm{u}(1)$, the mixed u(1)–gravitational condition and the three Yukawa requirements yields

$$Y_Q = q, \quad Y_{u^c} = -4q, \quad Y_{d^c} = 2q, \quad Y_L = -3q, \quad Y_{e^c} = 6q,$$

which at $q = \frac{1}{6}$ is $\frac{1}{6}, -\frac{2}{3}, \frac{1}{3}, -\frac{1}{2}, 1$: the observed assignment[P14R12]. *And the cubic condition* $[\mathrm{u}(1)]^3$ *is then satisfied identically rather than imposed*—the coincidence usually remarked on in the Standard Model's hypercharges is a consequence of the linear conditions once the multiplet structure is given.

So the undelivered content is two items and not three. What remains genuinely outstanding is the gauge group and the multiplet structure; *hypercharge follows from them by a condition this construction supplies. The per-wall character is what does the work here and is worth marking*: cancellation *in total* across three generations would not fix the assignment within one, and it is the separateness of the three walls that makes the condition apply generation by generation. *And the identification is more useful than a shared debt, because the cosmological end carries measurements and this end does not.* The two data are of different kinds and their quotient is therefore a number: from $\eta = n_b/n_\gamma$ one has $\rho_b/\rho_\gamma = (30\zeta(3)/\pi^4)\,\eta\, m_b/T$, and from the onset condition one has ρ_m in units of ρ_γ at the same temperature, so *the ratio of the matter's energy to the baryons' is fixed by measured quantities alone*—of order a few at the onset, and rising with the onset temperature. *That is an empirical statement about what the handover delivers*, and it is owed an account by this sector rather than by a component introduced to meet it. The same holds on the relativistic side: what shares the radiation with the photons is likewise this sector's to say.

Read this way the constraint proved in [6] is informative rather than prohibitive. It does not say that a geometric route to the representation content is impossible; it says that the connected isometry route is closed and identifies the single component the obstruction cannot reach. *Together with the numbers above, the two bound the search from opposite sides*—one fixing what an admissible mechanism must produce, the other fixing where such a mechanism may live. *We claim no construction here.* We record that the sector's undelivered content is not unconstrained: it is constrained by a proved structural result and by measured ratios, and the fair reading of that pair is a well-posed problem rather than a wall. Reversing the wall reverses $\chi_\pm$, flipping the chirality—the parity is the chirality, exactly as the algebroid paper reads it off the constraint algebra.

3 Three walls: the count

The single wall of Section 2 sits at $r = 0$, and $r = 0$ is not the throat's centre but a point *on* the throat circle—the back, $X_1 = -\alpha$, fixed relative to the hinge about which the slicing plane swings [7]. The substrate carries three such hinges, 120° apart at transverse distance 2α, of which the throat circle is the incircle. That distance is not a stipulation the fermion construction inherits but an output of the substrate's own geometry: the hole determines exactly one circle without further choice—the one on its own edge through its own centre—and 2α is where that circle ends, with the throat-as-incircle, the 60° the hole subtends there, and the triangle's nine-point circle being the throat all equivalent to it [7]. So the three walls of this section stand on a triangle the hole placed, and the $\mathbb{Z}_3$ that permutes them is the hole's own three-foldness rather than a choice made here. The walls therefore sit on the same circle everything else in the construction is read off—the circle whose radius is the curvature, whose tangents are the null rulings, and around which the slicing's lap runs—so the generation count this section obtains is one of the faces that circle carries, and is collected as such [5]. The $\mathbb{Z}_3$-fixed centre carries no wall; the walls lie on the circle, and the three hinges' walls are three distinct points on it (polar $180°, 300°, 60°$), a $\mathbb{Z}_3$-orbit (Figure 1).

That the centre carries no wall is not a further observation about the figure but a consequence of what a wall is[P14R38]. Vantage j's signed areal radius $r_j = \alpha \sin(\varphi - \theta_j)$ on the throat circle is the restriction of a linear form on the transverse plane—the coordinate perpendicular to hinge j's static pair—and the three forms have rank two, so their common zero set is the single point at the centre. A wall is a locus across which one vantage's superpotential changes sign, hence codimension one; a point is codimension two and has no side to cross to. The wall is accordingly the *line* $r_j = 0$, which is hinge j's own axis and meets the throat circle twice, at the front and at the back; naming $r = 0$ at the back fixes a point on one wall rather than selecting between two walls, and the two crossings are exchanged by the antipodal map, which is R. Taking the three forms' mutual angles—which is a different question from where they vanish—they are of equal length, 120° apart, sum to zero identically, and have mutual cosine $-\frac{1}{2}$: the weight system of the fundamental of $\mathfrak{su}(3)$, and in rank two nothing else has that shape. The transverse plane is therefore the weight plane and the vantage index the weight label, reached here from the metric geometry with no representation theory in the derivation. The empty centre is then the origin of that plane, and the fundamental has no zero weight: a state seated at the centre would be a zero-weight vector of **3**, of which there are none. The emptiness is an absence of a weight, not a vacancy waiting to be filled.

Two constructions are geometrically available: one plane on one chosen hinge, or one plane on each. They are not on equal footing.

Proposition 2. *Within CR the three-plane construction is forced.*

Argument. A one-plane construction must select *which* hinge to build on: a free modulus, unfixed by the geometry, an arbitrary choice among three $\mathbb{Z}_3$-equivalent options. The $\mathbb{Z}_3$-symmetric three-plane construction is the unique configuration carrying no such modulus—the symmetric point, pinned by the symmetry. CR is defined by maximal symmetry read as least-arbitrariness: the substrate is dS_5 precisely because a less symmetric choice would be unforced [5, 11], and the geometric-core paper extends this to the discrete sector explicitly, holding that the discrete breaking is itself maximally symmetric. A construction carrying an unfixed arbitrary modulus is exactly what that principle excludes. The one-plane truncation is therefore not admissible within CR, and the three-plane structure is selected. □

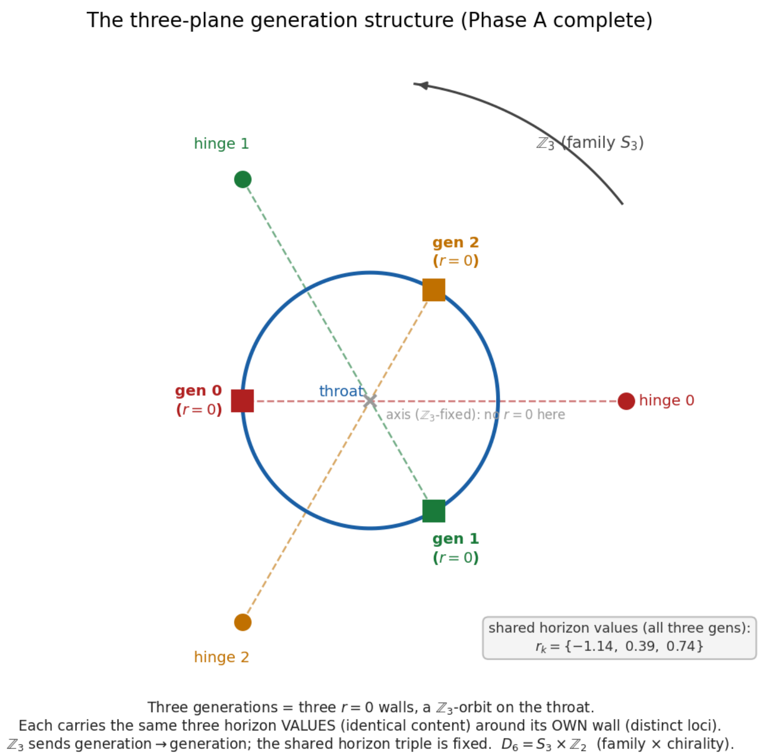

Figure 1: The three-plane fermion structure. Three hinges (radius 2α, polar 0/120/240) and their three $r = 0$ walls (on the throat circle, polar 180/300/60, each antipodal to its hinge)—a $\mathbb{Z}_3$-orbit. Each plane carries the same three horizon *values* (identical content) around its own wall (distinct loci): three identical generations. The centre (the $\mathbb{Z}_3$-fixed axis) carries no wall. $\mathbb{Z}_3$ sends generation to generation; $D_6 = S_3 \times \mathbb{Z}_2$.

The qualification is genuine and we state it plainly: Proposition 2 forces the count *within CR*, resting on (i) the maximal-symmetry principle, which is the programme's definition rather than an added hypothesis, and (ii) that the fermion generations are set by the matter construction at all—Proposition 1 makes this concrete, but the link is itself the claim the sector establishes. It is not a free-standing theorem. Inside CR it is forced. Two remarks sharpen this. The two legs are in fact one: given the three-hinge leaf, the three throat walls are *distinct* loci, so the wall-bound zero-modes have disjoint support and span a three-dimensional space—a flavour triplet, three physical states in any basis, not one state redescribed—so leg (ii) follows from leg (i) rather than standing beside it. And the pillar that remains, leg (i), is the programme's own *criterion of necessity*—Rule 2 of [12] in the ontological register—for which a symmetry-breaking modulus (which of three $\mathbb{Z}_3$-equivalent hinges) is exactly the adjustable parameter it rejects, and maximal symmetry the structure that requires its configuration. "Forced within CR" is thus forced by that vindicated criterion, not an added axiom; the honest edge is that a framework declining the criterion reads the natural single-hinge index—one—not three. The direction of that dependence is worth stating, since [12] in turn cites this sector: the epistemic paper *establishes* Rule 2 from the historiographic record, independently of any result here, and cites the three-plane selection only as a worked instance of the rule in the ontological register. The dependence is one-way and the mutual citation is not circular—this paper applies a criterion the other grounds elsewhere.

By Proposition 1 each of the three walls binds one chiral zero-mode; the three are localized at distinct walls, hence linearly independent. The generation count is the number of such modes, a wall-localized index insensitive to the non-compactness of dS_5 that obstructs a bulk index [6]: the modes are normalizable (in the leaf norm of Section 2) and pinned to the throat, and there are three. That count is a well-defined index in the precise sense CR's ontology makes available. In the leaf's proper measure $d\ell = dr/\sqrt{|f|}$ the closed slicing has *finite* total length[P14R10]—the horizon turning points at finite proper distance, the $r = 0$ crossing an integrable $\sqrt{\ }$-singularity—so the leaf is compact and the Dirac operator on it carries a *finite* analytical index $\dim\ker$ [20], exactly where the bulk index on the non-compact spacetime (tortoise measure) is obstructed. *The leaf is closed as well as compact*, and that is a second thing the measure delivers: with no boundary there is no Atiyah–Patodi–Singer correction and no η-invariant term, so the index is the interior one entire and the count is the whole of what the theorem returns.[L19] *Compactness delivers finiteness and not canonicity, and the distinction is load-bearing here.* In the proper radial coordinate of the leaf frame, $\mathrm{d}x = \mathrm{d}r/\sqrt{f}$, the first-order pair integrates in closed form: $\int W\,\mathrm{d}x = \int(\lambda\sqrt{f}/r)(\mathrm{d}r/\sqrt{f}) = \lambda\ln|r|$, so the zero mode is a power law $\psi = |r|^{\pm\lambda}$ rather than the tanh model's exponential. Near the branch point $f \to -2M/r$, so the leaf measure behaves as $\mathrm{d}\ell = \mathrm{d}r/\sqrt{|f|} \sim \sqrt{|r|/2M}\,\mathrm{d}r$ and normalizability of $|r|^s$ requires $s > -\frac{3}{4}$. The decaying branch $s = +\lambda$ satisfies this for every λ; the growing branch $s = -\lambda$ would require $\lambda < \frac{3}{4}$, which no $\lambda = j + \frac{1}{2}$ attains. At the horizons the measure carries an integrable inverse square root and ψ is finite, so the[7]. *One numerical coincidence is worth disowning before anyone builds on it*: the threshold here is $\frac{3}{4}$, and so is the essential-self-adjointness threshold the companion canonical paper meets at $a = 0$ [16]—but they are different computations. That one comes from $\sqrt{\gamma + \frac{1}{4}} = 1$ on exponents $\frac{1}{2} \pm \nu$ in $\mathrm{d}x$, an exponent gap of two; this one from $-2|\lambda| + \frac{1}{2} = -1$ on a density in $\mathrm{d}\ell$, a gap of three. *Different measures, different gaps, the same number by arithmetic and not by structure. Nothing in the count depends on a boundary condition*, the three modes having disjoint support and definite chirality either way. Each wall's mode is a definite chirality eigenstate $\sigma_y = +1 = R = \gamma^5$ (Prop. 1, the conjugate branch rejected), and the signed-radius flip is through the $r = 0$ *branch point*[P14R49] rather than a single-valued crossing on a loop [7, 13]—so the even-crossing constraint that binds a single-valued sign function on a circle does not apply, and the three same-chirality modes on disjoint support have no cancelling partner. Hence $\dim \ker_+ = 3$, $\dim \ker_- = 0$, and the count is the net chirality: a γ^5-graded index. Index-theoretic stability under deformations preserving the three-wall structure is the expected behaviour of such a graded count and is traced rather than computed here ([P14R1] establishes the mechanism—one mode per wall, the even-crossing obstruction on a simple loop, and the necessity of the $r = 0$ branch point—and marks the Atiyah–Singer statement on the branched bead as traced); we state it at that weight[L13]. *And it is worth saying which half is traced, because the two halves are not equally owed*: the *analytical* index is $\dim \ker_+ - \dim \ker_-$ by definition, so the count above is computed and needs no theorem; what the index theorem supplies is its equality with a topological integral, and what that equality buys is exactly the deformation invariance traced here. *So the citation is honest rather than load-bearing for the count*, and the corpus's naming follows the same division—*analytical index* appears and *topological index* never does[P14R16]. It is a leaf (vantage) index, which is why it is well-defined precisely where the spacetime bulk index is not.

$$\boxed{\text{number of chiral generations } = 3.} \tag{3}$$

What the count fixes about the dimension. The count above is read off the horizon cubic through the three hinges, and that cubic is an output of the four-dimensional metric function rather than an assumption of this sector. The constraint therefore runs the other way from the way it is usually posed: *the flavour content constrains the dimension*, rather than being handed one.

In D-dimensional Schwarzschild–de Sitter the mass is the slicing-dependent factor $2M = r_0^{D-3} - r_0^{D-1}$ in the gauge $\alpha = 1$, and two conditions this construction already imposes then bear on D. The first is the collapse. The family is parametrised by the sky angle, and the horizon relation is required to linearise there to a *single* multiple-angle—in the companion paper's words, the slicing scale $2/\sqrt{3}$ is "the one scale removing the residual harmonic" [7].

The two powers present are $D-3$ and $D-1$, so the harmonics standing below the top one number exactly *one* at $D = 4$ and $D = 5$ and *two or more* from $D = 6$ upward, while the construction has one scale to spend: from six dimensions up no slicing scale collapses the relation, there is no forced fold, and the family carries no generation count to read at all.

At $D = 5$ the collapse does occur, at scale 1, and returns a four-fold.

The second condition is the parity, and it separates those two. The mass function is odd in the signed offset exactly when D is even, so at $D = 5$ the orientation parity $r_0 \mapsto -r_0$ *fixes* each geometry rather than exchanging it with its conjugate: there is no mass-reflection $\mathbb{Z}_2$, hence no $\mathbf{3} \oplus \bar{\mathbf{3}}$ Nariai hexad, no outer factor of $\mathrm{Aut}(A_2) = S_3 \times \mathbb{Z}_2$, and no γ^5—which is that parity's Clifford generator on the cut (Section 5), and on which this paper's chirality rests through a superpotential odd in the signed radius (Section 2). A five-dimensional spacetime would therefore carry four generations and no handedness—and, *on the same computation, no antimatter either*, since the parity whose absence removes γ^5 is the one whose absence removes the antifundamental.[P14R36] *The two deliverables are one fact seen twice, and it is visible in the collapse itself*: at $D = 4$ the relation collapses to a pure multiple angle $2M = \frac{2}{3\sqrt{3}} \sin 3w$ of zero mean, whereas at $D = 5$ it collapses to $2M = \frac{1}{8}(1 - \cos 4w)$—one harmonic *plus a constant*.

Both are single-harmonic in the sense the collapse condition asks for, but the constant is not nothing: it holds $2M \geq 0$ across the whole swing, so the mass never changes sign, and the sign change is the entire content of the mass reflection. *And the same obstruction is visible a third time, in the root set itself, where it exhibits its mechanism.*[P14R42]

Writing the horizon condition with the slicing's own root substituted, $P_D(r) = r^{D-1} - r^{D-3} + r_0^{D-3} - r_0^{D-1}$, the factor $(r - r_0)$ divides at every D—that is the designation split—and $-r_0$ is a root *exactly at odd D*, since substituting it returns $2 \cdot 2M$ at even D and 0 at odd.

So at $D = 5$ the cofactor degenerates to $(r + r_0)(r^2 + r_0^2 - 1)$, a line and a circle, and the $1 + 3$ is really $1 + 1 + 2$ with the extra one being $-r_0$ itself, where at $D = 4$ the cofactor is a genuine conic and at $D = 6$ and $D = 8$ it does not factor at all. The root-set condition and the mass-parity condition are exact complements dimension by dimension, and the reason is structural: at even D the reflection $r_0 \mapsto -r_0$ carries the geometry to a *different* member of the family, which is what lets it serve as a matter/antimatter parity, while at odd D the reflected root already lies in the same geometry's own root set, so the parity has nothing to relate. *The geometry is its own image, and that is why five is vector-like. And two further structures fail at odd D for the same evenness, the three being one fact.*[P14R43]

The polynomial $r^{D-1} - r^{D-3} + 2M$ has its two powers of one parity, so it is even in r exactly at odd D; its root set is then stable under a fixed-point-free involution, and the monodromy over the mass plane commutes with that involution.

At $D = 5$ this confines the monodromy to the centraliser of $(0\,1)(2\,3)$ in S_4, of order eight against twenty-four, so the monodromy group cannot be the full symmetric group on the roots and the four-dimensional statement that it is has no odd-dimensional analogue *That is an upper bound, and the contrast it draws needs the bound attained*: "contained in a group of order eight" is also consistent with a trivial monodromy, which would leave the five-dimensional deck structure empty rather than merely smaller. Continued around loops in the mass plane the permutations generate a group of order exactly eight at $D = 5$—the full centraliser, the imprimitive symmetry of two $\pm$ pairs—against the full S_3 at $D = 4$, so the confinement is a real structure and the contrast is between two non-trivial groups rather than between a group and nothing[P14R54L13]..

The root set is likewise not of A_2 type: the roots sum to zero at every D, but at four that is three roots summing to zero, and at five it is four summing to zero *in* $\pm$ *pairs*—so at five there is no A_2 whose automorphism group could factorise or fail to. *The constant and the evenness are the same obstruction read in two variables*—and with no sign change there is no antifundamental, hence no hexad, hence none of the twelve designations the four-dimensional dial carries. Among the dimensions in which the count exists at all, four is the only one that also carries a chirality, so the two halves of this sector's own delivery—the number and the handedness—are jointly available at exactly one dimension[P14R50]. Both conditions are read through the sky angle and so through the gnomonic chart, and that chart is not a four-dimensional import: the companion paper forces it by the straight-line criterion alone—a faithful planar chart must carry the straight line of sight to a straight line, and the gnomonic projection is the unique projection sending every great circle to one—"independently of the observer or the slicing scale" [7], a property of S^n in every n, and the observer's celestial sphere in D dimensions is S^{D-2}. (That paper's *secondary* exclusion of the orthographic, which turns on $\sin\theta \leq 1 < 2/\sqrt{3}$, is specific to the four-dimensional scale and is not used here.) The altitude is the same as the rest of this section, and the one assumption is named: the D-dimensional metric function is taken to be the standard Tangherlini–de Sitter one, an extension of the codimension-one operator rather than a rederivation of it at general D. Within CR, and at that weight, the observed flavour skeleton is not merely consistent with four dimensions but selects them.

3.1 The correspondence, stated in Standard Model terms

Because the papers above speak of "generations" without saying what a generation contains, we set the correspondence out explicitly. A reader who knows the Standard Model should be able to see at once which of its structures this sector fixes and which it does not.

What a Standard Model generation contains. In left-handed Weyl form, one generation is five irreducible multiplets of $\mathfrak{su}(3)_c \times \mathfrak{su}(2)_L \times \mathrm{u}(1)_Y$:

$$Q = (u, d)_L : (\mathbf{3}, \mathbf{2})_{+1/6}, \quad u^c : (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}, \quad d^c : (\bar{\mathbf{3}}, \mathbf{1})_{+1/3}, \quad L = (\nu, e)_L : (\mathbf{1}, \mathbf{2})_{-1/2}, \quad e^c : (\mathbf{1}, \mathbf{1})_{+1},$$

totalling $6+3+3+2+1 = 15$ Weyl fermions, or 16 with a right-handed neutrino $\nu^c : (\mathbf{1}, \mathbf{1})_0$[P14R21]. The quark/lepton distinction is exactly the first entry of each triple: a quark carries the colour $\mathbf{3}$, a lepton the singlet $\mathbf{1}$. Three generations is 45 (or 48) Weyl fermions in all.

What this sector fixes. Three features of that skeleton, and only three.

- *The number of generations is three*, as a γ^5-graded index of wall-localized zero-modes ($\dim \ker_+ = 3$, $\dim \ker_- = 0$; Section 3).
- *Each is chiral*, carrying a definite γ^5 eigenvalue (Proposition 1). This is not decoration: the Standard Model's content is chiral, and a construction returning a vector-like spectrum would be excluded by that fact alone.
- *The three are related by a global S_3* (Section 4)—a family symmetry, not a gauge symmetry.

What it does not fix, and the list is longer. The internal content of a generation is untouched: how many Weyl fermions it holds (15 or 16), their $\mathfrak{su}(3)_c$ representations—hence *which of them are quarks and which are leptons*[P14R20]—and their $\mathfrak{su}(2)_L$ doublet/singlet structure. *The hypercharges are not a third undelivered item*: given those two, they follow from the per-wall anomaly conditions of §2.1 together with the existence of the Yukawa couplings, uniquely up to normalisation. Anomaly cancellation, which the Standard Model's fifteen satisfy non-trivially and which is among the sharpest constraints on any proposed content, is not addressed. Neither is the mass spectrum, the mixing, nor the existence of ν^c. *The zero-mode of Proposition 1 is one Weyl mode per wall, not fifteen*: the index counts generations, not the states within them.

Two consequences of stating it this way are worth drawing. First, the correspondence is $3 \leftrightarrow 3$ at the level of *families*, with every representation-theoretic fact about the Standard Model on the far side of the wall of [6]; the sector is compatible with the Standard Model's content in the weak sense of not contradicting it, and does not predict it. Second, the global S_3 is a point of contact rather than a free addition: the Standard Model has no exact family symmetry—the Yukawa couplings break it badly—so an exact S_3 delivered here must be broken by whatever supplies the masses, and the manner of that breaking is a constraint this sector inherits rather than a freedom. *We record that as an obligation, not a result. That obligation can be checked against a literature rather than left standing, and it is worth doing because the check could have gone badly.* S_3 is among the most studied discrete flavour symmetries: the $S_{3L} \times S_{3R}$ proposal predicting quark-sector flavour democracy has been followed by an extensive body of work on how to break that democracy and recover realistic spectra and mixing [2, 3]. The structure of the symmetric limit is definite and matches what a three-object permutation representation gives: the mass matrices take a universal two-term form, one piece proportional to the identity and one to the democracy matrix. On that decomposition the observed pattern is not accidental—the charged-fermion matrices are strongly hierarchical, the quark mixing matrix is close to the identity, and the lepton mixing matrix carries two large angles, all as consequences of which term dominates where [3].

Two things follow for this sector, and they pull in opposite directions. The obligation is met in the sense that matters: an exact S_3 broken by the mass mechanism is a viable and worked framework, not an excluded one, so the sector is not committed to a symmetry the data refute. But it is not met for free. Realistic fits generally augment the group—$S_3 \times Z_3 \times Z_6$ in one construction, where the neutrino sector's breaking supplies a naturally small solar-to-atmospheric ratio and next-to-leading corrections bring θ_{13} into the measured range while the quark sector accommodates the

Cabibbo angle and the differing up- and down-type hierarchies [4]—and the breaking pattern does substantial work. *What this sector supplies is the symmetry, not the breaking*: the S_3 arrives here from the substrate's geometry rather than being posited, which is a point of contact worth having, while the breaking that generates the observed masses and mixings remains external to it, in the same place as the gauge representations. The mass spectrum and the gauge representations are external to it and are so marked. This is the innermost of the three registers of maximal symmetry the geometric-core paper draws together [5]: the substrate maximally symmetric, the evolving space maximally symmetric, and now the discrete matter structure maximally symmetric and counted.

4 The family symmetry

The three walls are permuted by the symmetry of the hinge triangle. The $\mathbb{Z}_3$ rotation cycles them ($180 \to 300 \to 60$); each of the three reflections—across the axis through one wall—swaps the other two and fixes it. The three transpositions and the three-cycle generate the full S_3. With the chirality $\mathbb{Z}_2$ of Proposition 1, the discrete group carried by the sector is

$$D_6 = S_3 \times \mathbb{Z}_2 \qquad \text{(family} \times \text{chirality)}, \tag{4}$$

the automorphism group $\mathrm{Aut}(A_2)$ already identified as the substrate's discrete structure [7, 8, 9].

Writing it as a product invites a question the product hides: D_6 has ***three*** *subgroups of order six, and which one the sector carries is a physical statement rather than a labelling.* Exactly one is cyclic, $\langle \text{the Weyl 3-cycle} \rangle \times \langle \gamma^5 \rangle$—*and it is worth saying that this is* ***not*** *the family symmetry paired with chirality*, since the $\mathbb{Z}_3$ inside $\mathrm{Aut}(A_2)$ rotates the three horizon *roots* and is therefore the within-state index of §6.1, while the generations' own $\mathbb{Z}_3$ is the turnaround's deck, which fixes the horizon cover's base and never permutes its fibre and so is no subgroup of this group at all—and the other two are *both* isomorphic to S_3 while being different subgroups: $S_3 \times 1$, in which the roots are permuted and the chirality untouched, and the graph $\{(\sigma, \operatorname{sgn}\sigma)\}$, in which every root transposition carries a chirality flip. *The boundary paper's own reading of the hexad settles which*: there σ exchanges a pair *within* a sign-half while the backward-radial reflection—the outer $\mathbb{Z}_2$—is what carries between the halves [6], so a transposition carries trivial $\mathbb{Z}_2$ component and the family symmetry is $S_3 \times 1$ and not the twisted graph. *Root transpositions do not flip chirality*, which is why the three walls can be same-chirality at all (§3).

Two further facts about this group are established elsewhere and bear directly on the sector.

First, the group is larger than D_6, and the excess is substrate-derived. The residue pairing that the horizon roots' surface gravities put on the root triple has a holonomy about the Nariai points—the Klein four-group of even sign changes, arising as the per-root resolution of $\sqrt{\Delta}$, the same square root whose non-squareness fixes the Galois group as S_3 [8]. Adjoined to the walls' S_3 it closes $W(A_3)$ in its Weyl embedding, and with the chirality parity a group of order forty-eight; and A_3 is the root system of $\mathfrak{so}(6, \mathbb{C})$, the complexification of the substrate's own isometry algebra, with A_2 inside it by deleting one node [9]. *So the sector's discrete group is the substrate's own, where this section has been reading a sub-root-system of it.*

Second, and physically, that holonomy acts on the walls' chiralities and protects the count's parity. By Proposition 1 each wall's chirality is a definite σ_y eigenvalue whose sign is the sign of the signed-radius flip, so a sign change is a reversal of that wall's chirality; the holonomy's elements change signs in *pairs*, unimodularity excluding every odd change. *The observed configuration is accordingly the unique member of its orbit with all three walls at one chirality*, and $\dim\ker_-$ cannot be reached from $\dim\ker_+ = 3$ by any number of loops, three and zero lying in different parity classes[P14R14]. *The count is thus not merely what the index returns but what no holonomy of this* *connection can move. Two guarantees are in play here and are worth keeping apart*: compactness of the leaf makes the count *defined*, and what makes it *stable* is the wall's spectral gap—its bound modes are separated from the continuum by the full asymptotic mass, so no small deformation of the wall carries a state across the threshold and changes the number.[L19]

That the family symmetry the three walls carry is *global* rather than gauged is not a choice made here but the *kind* the discrete structure fixes: the Weyl S_3 is the monodromy symmetry of the solution space and no substrate isometry, whereas the orientation parity $\mathbb{Z}_2$ *is* an isometry, acting on the cut spinor as γ^5—so chirality descends gauged and flavour global, the Standard Model's own arrangement following from which factor is an isometry [8, 9].

That chirality rides the *discrete* factor is moreover forced, not merely sorted: a chirality carried by a connected-group isometry would be rendered vector-like by the Atiyah–Hirzebruch index obstruction, so an observed chirality can live only on the discrete orientation parity that obstruction cannot reach [6].

Each plane carries the same three horizon values, so the three generations are identical in content, distinguished only by their wall; this is why the family symmetry is a symmetry of *identical* copies, the defining property of Standard-Model generations. And the wall is not a bare locus. Each hinge *designates* one of those three roots as its own black-hole horizon, so a transposition of roots is a hop to a neighbouring hinge, and the Weyl S_3 *is* the relation among the three hinges rather than an abstract relabelling [7]. The S_3 the three walls carry and the Weyl S_3 of the root system are therefore *one group*, not two that happen to agree in order: a generation is the vantage that takes its own root as its hole, and which root it takes is the same distinction as which wall it binds at—the hinge fixes both. This is what makes "one object read three ways" a statement about the sector rather than about the cubic alone: the three readings are the three hinges, and the fermion sector is where they become three states. At the massless level the three are degenerate and S_3 is exact, and this degeneracy is *forced*: the three shared horizons are roots of one cubic $r^3 - r + 2M = 0$, and every root, designated the slicing parameter, returns the *same* $2M = r_0 - r_0^3$ (as $r^3 = r - 2M$ for each), so the three carry one mass parameter—one object read three ways on the fully D_6-symmetric horizon locus, a diagonal line and a $45°$ tilted ellipse in the (r_0, r)-plane [7], on which the 2+1 of a designated root against the ellipse pair is the diagonal designation σ and the sign 2+1 (two horizons of one sign against one) the reflection through the origin ($2M \mapsto -2M$, the chirality parity), each a description structure paired with its symmetric twin—so no asymmetric handle sits in the mass *parameter*. The locus does carry a flavour-*breaking structure* (the slicing singles one root against the ellipse pair), supplied with the triplet in the boundary paper [6]; but the mass *values* are the ordinary electroweak route, and whether that geometric structure is the physical hierarchy is not claimed—outside the present scope.

5 The two factors: why they differ in kind

L6

The sector's two data are a chirality and a count, and Section 4 has just written them as the two factors of one group, $D_6 = S_3 \times \mathbb{Z}_2$ (family $\times$ chirality). Two things about that group have been established elsewhere and are used above: that its factors act on independent structures—the Weyl S_3 on the three horizon roots and the inversion on the two null rulings, the inversion central [9]—and that they descend differently, one gauged and one global. *The product's being direct is worth separating from both, because it is forced and therefore carries no information about this substrate*: $\mathrm{Aut}(A_2)$ is a Weyl S_3 extended by a diagram $\mathbb{Z}_2$, every automorphism of S_3 is inner, and a semidirect product by an inner automorphism is isomorphic to the direct one—so there was never a semidirect alternative for the geometry to have excluded.

What the geometry supplies is which relation each factor bears to the substrate's waist, and that the two differ in kind; the group structure follows and does not evidence it.

Both have been read off the constraint algebra and off the light cone [9]. *And the chirality factor has a second realisation, in a sector with no fermions in it, which is worth stating because it shows the parity is the substrate's and not this sector's.* In the radiating sector the same $\mathbb{Z}_2$ appears as the graviton's two helicities: chirality there is the turning of the polarization plane, the sign of the rate at which the polarization angle turns, *absent* wherever a residual connected isometry pins the polarization to a fixed axis—the swept SO(3) of the symmetric sector completing a reflection into a rotation and identifying the two handednesses, a mirror and not a chirality—and genuine once that swept rotation is lost [15]. *That is the same mechanism operating here, by a different instrument.*

What makes the wall's bound mode chiral rather than mirror-paired is that no connected group identifies its two handednesses, which is what the index obstruction of [6] says of a connected gauge action and what confines the parity to the disconnected component. *In the radiating sector the identification is lifted by losing a residual isometry and the matter is a direct computation; here it is barred by a theorem about connected groups*—one parity, two sectors, two instruments. The corroboration runs the way that matters: a sector with no fermion content returns the same $\mathbb{Z}_2$, so the factor read off the light cone is a feature of the substrate rather than an artefact of the wall construction. A third route reaches the same factorisation from the numerals alone, and reaches it without being told to look: an audit of the recurring numbers of the construction, asking of each pair only whether one object derives both, returns every weld through one of these two factors—the triple-angle map for the three, the twelve, the thirty, the $\sqrt{3}$ and the $\sqrt[3]{2}$; the ruling-exchange for the two—and every proved split across them, so that the numbers weld *within* a factor and split *across* it.

The audit, its receipts, and the splits that cut its law are collected in the programme's combinatorics ledger (`COMBINATORICS_LEDGER.md`; P14R5, P14R6, P14R7), alongside the figure programme's. Read on the substrate's waist they are one fact, and this is the paper to state it in, because this is where the two factors become matter: the $\mathbb{Z}_2$ as a bound state's chirality eigenvalue (Proposition 1), the S_3 as a count of three (Section 3). Because that $\mathbb{Z}_2$ is a substrate isometry it grades a chirality in the radiative sector as well as this one—the graviton's two helicities [15] and the fermion's γ^5 here are the one gauged factor's two physical faces, the same $\mathbb{Z}_2$ read in the two sectors, settled there by direct computation on the free wave and here by the domain-wall zero-mode. Its radiative face becomes a *genuine* chirality only past the wall of the symmetry-reducible sector, where the last swept SO(3) is lost and no isometry survives to turn the exchanging reflection into a rotation—the geometric onset of the parity the fermion sector here carries as γ^5 [17].

The two structures the factors act on are the two relations a figure can bear to a circle. The three roots are the three special points *on* the throat circle: an offset line meets it at two points A and B, with $r = 0$ fixed at the back, and those three points are the geometric counterparts of the three roots of the horizon cubic [7]—which is why the walls of Section 3 lie on that circle and not elsewhere. That offset *is* the mass—matter as the bend of the cut, G entering only as the offset-length—so the same offset furnishes both factors geometrically: its odd offset–mass relation ($2M \mapsto -2M$) is the chirality parity, and its three roots the count [10]. The two rulings are the lines *tangent* to it: the substrate's null rulings *are* the tangents to its waist, "doubly ruled by straight null lines" and "every tangent to the throat is null" being one statement, the power of a point with respect to that circle being the square of its height [5]. So the automorphism group factorises because *on* and *tangent* are independent relations to one circle; the direct product is that independence. That there is a second factor at all is itself contingent: the inversion is an *outer* automorphism—not already inside the Weyl S_3—only because the cubic realises A_2, the one rank-two system with $-1 \notin W$; were negation inner (as for A_1, B_2, G_2) $\mathrm{Aut}(A_2)$ would collapse to the Weyl group and leave no chirality factor to sort, which the groupoid paper marks the condition of there being two factors at all [8]. The on/tangent independence is then why the two that do exist form a *direct* product.

And the factors differ in *kind* for the same reason, which is why the difference is not an extra fact requiring its own argument. A tangent is a line *of* the substrate, so exchanging the two rulings is a motion of the substrate—an isometry, acting on the cut spinor as γ^5, and the chirality descends *gauged*. A root *labels a different cut*, so permuting the three moves through the solution space and is no motion of the substrate—the monodromy symmetry, and the family symmetry is *global*. The same distinction is what the algebroid paper reads off the light cone, in the same objects: the sky-angle periodicity is two steps along a null ruling, hence a closed path of light and an isometry, whereas the Weyl reflection is a loop in the complex $2M$-plane, and $2M$ labels the family rather than pointing along the manifold [9]. The two readings are one reading, because the ruling and the tangent are one line.

The Standard Model's arrangement of a gauged chirality against a global flavour is therefore, in this reading, the difference between being *on* the one circle and being *tangent* to it—and the present sector is where that difference becomes two physical data: a bound state's chirality eigenvalue, and a count of three. Two qualifications keep the statement at its weight. It explains the *kind* of each factor and not the gauge content, which is excluded from the isometry [6]; and it is a reading of structure already established rather than a new derivation—what it adds is that the two structures, and hence the two kinds, are one circle's two relations, so that the arrangement follows from the geometry having a waist rather than from a further principle.

6 The cosmogenesis of the generations

The generations are wall-modes of the existent leaf (Section 3), and CR's cosmogenesis carries that leaf across the branch point at $r = 0$, where collapse continues as cosmology [6, 11]. Three features of the crossing fix what becomes of the generations, each read off established structure. *Inheritance:* the causal reassignment fixes the expansion rate and does not touch the leaf-carried content, so the generation structure—being leaf-carried—is inherited across the branch point unchanged *on the conjectured rate-only reassignment*, a placement the corpus has not derived, the fermion-sector reading of the corpus's rate-reassigned, matter-inherited law.

This is the matter-sector face of the layered reading the cosmology computes on: the reassignment acts on the *lapse*—the stacking rate, which it resets to the geometry-set law—and not on the *leaf*, whose bend is the content [10, 11]; so the generations, being leaf-carried, cross as inherited content exactly as the radiation and matter densities do, the fermion instance of the general fact that at the seam only the rate is reset and everything the leaf carries passes through. *A second argument reaches the same conclusion without the conjecture, and it is worth having because it does not depend on what the reassignment touches.* The walls are not carried *through* the crossing: they are *at* it.

The signed areal radius passes through zero at a point on the throat circle, which is why the walls lie on that circle and not elsewhere (§2), so the wall locus and the branch point are one locus. And the segment terminating there occupies no cosmic time at all—the real part of the complex cosmic time does not advance along it, so the crossing is a boundary of zero duration rather than an interval [16, 11]. *A structure localized at a boundary of zero duration has no interval in which to be altered*, whatever a reassignment acts on. The inheritance therefore does not rest on the reassignment being rate-only: even one that touched the leaf would have no elapsed time in which to touch it. *What the rate-only placement adds, if it holds, is that nothing acts on the leaf at all; what the zero-duration argument gives without it is that nothing could. Origin:* the Nariai crest [22] is the fixed point of the root-permutation, where two of the three roots merge (the discriminant $4 - 3r_0^2$ vanishing); the three generation-loci are S_3-symmetric there and split into three distinct generations undercritical, so the family structure *originates* at the crest and unfolds as the universe expands. *Chirality:* the crest is a fixed point of S_3 but *not* of R [6], so the parity $R = \gamma^5$ is untouched—the inherited matter stays chiral, and no matter/antimatter asymmetry is a cosmogenesis event; it stays the ordinary route.

This is rooted in the substrate's discrete geometry.

The chirality/mass parity $R = \gamma^5$ is the mass-reflection $r_0 \mapsto -r_0$ (equivalently $2M \mapsto -2M$), the orientation/diagram-automorphism parity, realised on the cut spinor as γ^5; as the A_2 diagram automorphism it carries the fundamental $\mathbf{3}$ to the antifundamental $\bar{\mathbf{3}}$ [7, 9], and on the wall mode it is an exact operator (Proposition 1). So the R-image of a generation is its antifundamental, and within CR that is its *antimatter*: the dual $\bar{\mathbf{3}}$ branch is antimatter at exactly the weight the fundamental branch is matter—forced within CR by the same discrete residue, with the world-correspondence of the identification the data's to judge as it is for the matter it pairs.

The discrete matter/antimatter skeleton—representation ($\mathbf{3}$ vs $\bar{\mathbf{3}}$), chirality (γ^5), and mass-sign (R-odd $2M$)—is what the geometry supplies on both sides of the pair; the charge structure that closes C is field-level on both sides equally, not a substrate datum—the charge entering the geometry only as the R-even Q^2, so charge conjugation is present there as a symmetry rather than an absence and is no substrate isometry, the outer $\mathbb{Z}_2$ of $\mathrm{Aut}(A_2) = D_6$ being R and not C [6]—so that geometry supplies matter's discrete skeleton and not its charge is no more an obstacle to naming the $\bar{\mathbf{3}}$ branch antimatter than to naming the $\mathbf{3}$ branch matter.

What R is *not* is the time reflection T: they are distinct reflections—R spacelike and horn-preserving, T timelike and horn-flipping, so $R \neq T$ on the substrate's A_2 hexad, established in the slicing paper [7].

That distinctness is exactly why the matter/antimatter parity is *not* a cosmogenesis (time) event: antimatter is the substrate's ordinary R-partner of matter, not something made at the seam, which the crest, a fixed point of S_3 but not of R, leaves untouched. Read on the cosmogenesis, this same R-partner branch is not abstract: it is the conjugate ($r < 0$) branch of the bead, the completed collapse whose future is our expansion, so the progenitor of our own universe sits on it—an antimatter black hole, in the signed-radius continuation [11].

The relation is therefore standing and relational (our matter and the progenitor's, the two ends of one R-conjugation across the bead), consistent with its being no seam-generated asymmetry. R is likewise distinct from the areal-radius spatial parity $r \mapsto -r$—the corpus's P—whose Clifford generator $\gamma^1\gamma^2\gamma^3$ anticommutes with γ^5.[1] The wall-mode's dynamics transitions with the reassignment: under the signature flip the hole-side spatial mass $W = \lambda\sqrt{f}/r$ passes into a cosmological-side temporal term (its turning imaginary past the horizons being precisely this), so the bound wall-mode

[1] Corpus convention: R is the orientation/mass-reflection parity, the A_2 diagram automorphism $\in \mathrm{O}(5,1) \setminus \mathrm{SO}_0(5,1)$, realised on spinors as γ^5, carrying $\mathbf{3} \leftrightarrow \bar{\mathbf{3}}$; P is the areal *spatial* parity $r \mapsto -r$ ($\gamma^1\gamma^2\gamma^3$, anticommuting with γ^5); T is the time reflection. The two parities are distinct, anticommuting operations, uniformly named this way across the corpus. *And one further convention, fixed here because its absence has cost more than the parities' did*: **"the wall" and "the branch point" denote the wall of the hinge under discussion.** There are three, one per hinge, at distinct points of the throat circle with disjoint support, and each vantage's signed areal radius vanishes at its own and at neither of the others [7]. The bare singular is used throughout because the natural sentence is about one of them at a time; *the test a reader should apply is that a sentence which would be false if there were three is a sentence about a single vantage and must name it.* This is stated because the sector's own winding computations had been read as though the wall were unique, which makes the branched cover cyclic and its monodromy abelian, and the whole of the colour structure is the difference between that reading and the correct one.[P14R34]

continues as a fermion propagating in cosmic time, and the three progenitor wall-modes become the three propagating families of the expanding universe, carried through faithfully by the null characteristic seam.

The explicit continuation of the exact zero-mode onto the cosmological leaf is carried out on a concrete undercritical model (receipt: P14R3): the superpotential $W = \lambda\sqrt{f}/r$ is real between the horizons, where the mode is the exponentially bound wall-state, and turns imaginary past a horizon ($f < 0$), where the same mode becomes a pure phase—a fermion propagating in the now-timelike radius; on the $E = 1$ expanding leaf $r(\tilde{\tau}) \propto \sinh^{2/3}$ this is the standard massless-Dirac form of conformal weight $a^{-3/2}$, *which is field-theory bookkeeping on a chosen chart and not a substrate structure*—the weight is the one a massless spinor carries on any Friedmann leaf, claimed here for the continuation and for nothing more[L8], so each of the three progenitor wall-modes continues term for term into a propagating family and the γ^5 chirality, untouched by the real-to-imaginary continuation, is preserved.

Inheritance, origin, chirality, and the bound-to-propagating character stand on the crossing's established null-characteristic machinery.

The universe those three families cross into is the one whose primordial light-element abundances the cosmogenesis paper computes [14]—the same branch-point crossing, read on the inherited radiation content rather than the fermion leaf, fixing the thermal history the abundances record.

As throughout, this is coherence within CR; what the world's matter is remains the data's to judge.

This realises, on the built sector, the positive closure the boundary paper draws [6]. There charge conjugation is shown to factorise on the cosmogenetic bead, $C = (Q \mapsto -Q)_{\text{field}} \circ (R \circ K)_{\text{geometric}}$, its kinematic (Feynman–Stückelberg) face carried geometrically by the composite of the mass-reflection R and the cosmic-time reality involution $K : \tilde{\tau} \mapsto \bar{\tilde{\tau}}$, only the electric-charge sign closing from the field. On the present sector that geometric face acts on the *actual* zero-modes: R carries each matter generation's wall-mode to the bound opposite-chirality mode on the reversed ($r < 0$, $2M < 0$) wall—its antimatter partner $\bar{\mathbf{3}}$, exact by Proposition 1—and K is the very real-to-imaginary seam continuation that carries the bound wall-mode to the propagating cosmic-time fermion above, so $R \circ K$ acts on the built modes as charge conjugation's kinematic face, the particle↔antiparticle relation of that conjugation being the two ends of this sector's R-conjugation across the bead (receipt P14R8). It is that face and no more, exactly as the boundary requires: the zero-modes carry the discrete flavour skeleton and not the gauge charge, so $R \circ K$ carries the discrete matter/antimatter conjugation while the electric-charge sign stays field-level—the identification of a mode with a *specific* charged particle, and the full antilinear C, remaining the ordinary route. The operator half is explicit and grounded—the reality-involution lift $S = \gamma^0\gamma^1\gamma^3$ gives $\gamma^5 S = -\mathrm{i}\gamma^2 = -(C\gamma^{0\top})$, implementing $\psi \mapsto \psi^c$ on the cut spinor (`storyboard_receipts/A3_spinor_lift.py`)—but it is an operator identity carrying no charge, so the matter/antimatter *pairing* of the two ends rests on the actual disjoint wall-modes of Proposition 1, not on the operator alone; absent that built construction the frequency-wing/species identification would remain unclaimed. The factorisation the boundary paper states is thereby realised on the sector it bounds, at exactly its earned weight, and no further: coherence within CR, the world-correspondence the data's to judge.

6.1 Which three the generations are seated on, and why it is not the hinges

The count above is read off the three walls and the family symmetry off the three hinges. *Those are one three, and it is not the generations' three*—which the substrate's null structure settles, from outside this sector and by a fact the hinge reading cannot reach.

The substrate's null structure admits a *bound* triple—a puncture of a hinge together with the two punctures its null generators reach—and the causal classification of the six hinge-ends forces such a triple to take one puncture per hinge: two ends of one hinge are timelike separated, two on one horn spacelike, and only the cross pairs null.[P3R49] A bound triple is therefore one-per-hinge without choice. If the three hinges were the generations, a bound triple would be one constituent from each generation, which is not what a bound triple of like constituents is. *So the hinge three is the index that distinguishes constituents within a bound state, not the index that distinguishes copies of the whole state*, and the generations must be seated elsewhere.

They have somewhere to go, and it is the construction's own second three. The framework paper establishes that no affine change of variable identifies the horizon roots with the roots of the comoving turnaround (*lem:twoturnings*); the marginally-bound congruence integrates to $r^{D-1} = 2M\alpha^2 \sinh^2\big((D-1)\tilde{\tau}/2\alpha\big)$ in D dimensions, and the shift $\tilde{\tau} \mapsto \tilde{\tau} + 2\pi i\alpha/(D-1)$ leaves r^{D-1} invariant while sending $r \mapsto e^{2\pi i/(D-1)}r$, so that second three carries a cyclic deck action of order $D-1$.[P3R33] Three consequences follow, and the third is the one that made the move admissible.

First, the count is unchanged and the dimension argument with it. Both threes are $D-1$, and for one reason: clearing denominators, $f = 0$ is a polynomial of degree $D-1$, and the two are the two things one does with that single polynomial—the horizon three is its zero set, the turnaround three the deck action of the integral of $1-f$. The fold is therefore a property of f and not of which structure reads it, so Section 7's conclusion—that four is the only dimension carrying both a count and a handedness—holds on either seat, its $D = 5$ member included.

Second, the family symmetry the generations bear is a different group from the hinges', and the difference is substantive rather than a matter of naming. The Weyl S_3 of Section 4 is the relation among the three hinges, and on this reading that is a statement about the within-state index: its transpositions are the hops between hinges that a bound triple needs in order to be a singlet. The turnaround three carries the cyclic deck action alone. So the family symmetry the generations bear is $\mathbb{Z}_3$, and $\mathbb{Z}_3 < S_3$ properly.

Third—and this is what had to be checked before any of it could be said—the chirality survives. The grading is the mass reflection R, and R acts on the turnaround branch: under $M \mapsto -M$ the right-hand side of the law above becomes non-positive for real $\tilde{\tau}$, so a real image exists exactly when $D-1$ is odd, and its image is $r \mapsto -r$, the signed radius. That condition is D *even*—the same condition Section 7 already reads off $2M = r_0^{D-3} - r_0^{D-1}$ being odd in the signed offset.[P3R32] The parity argument therefore transfers to the new seat without modification. Composed with the deck action it gives $\mathbb{Z}_6 = \mathbb{Z}_3 \times \mathbb{Z}_2$.

That is not the whole of what the turnaround carries, and the correction is worth making here because it runs the opposite way to what one would expect and does not disturb the argument above.[P14R45] It is natural to read the $\mathbb{Z}_6$ as an index-two subgroup of the D_6 the hinges carry, with the missing part exactly the transpositions the within-state index needs.

The transpositions are not missing. The time reflection acts on cosmic time as $\tilde{\tau} \mapsto -\tilde{\tau}$, and the outward law $r^3 = 2M\alpha^2 \sinh^2(3\tilde{\tau}/2\alpha)$ is exactly invariant under it, $\sinh^2$ being even—so T descends to the turnaround. But it does *not* commute with the deck action: it conjugates it to its inverse, $TDT^{-1} = D^{-1}$, which is the dihedral relation. Hence $\langle D, T\rangle \cong S_3$ and, with the parity adjoined, the turnaround carries D_6 of order twelve after all. *What keeps this from collapsing the distinction drawn above is that the two S_3's permute different triples*—the hinge one the three horizon *roots*, this one the three cube-root *sheets*—and the companion framework paper's *lem:twoturnings* establishes that no affine change of variable identifies the two root sets. *The statement is available in the stronger form the seating here actually wants*: the turnaround's deck carries a root of the horizon cubic to a root of the same cubic only at $r = 0$, since $(\omega r)^3 - \omega r + 2M = r(1-\omega)$ on the root variety, so it *fixes* the horizon cover's base and does not permute its fibre—and the two threes are therefore not related by any covering-space construction, rather than merely by no affine one[P14R4]. *Seating the generations on the turnaround rather than the hinges needed the two threes to be genuinely different objects*, and had the turnaround deck been a storey of the horizon cover's tower this subsection would have had no room to move.

So the hinge transpositions remain the within-state index's; the turnaround simply has transpositions of its own—*though whose transpositions they are is worth saying, because it decides what kind of family symmetry this sector delivers.*[P14R35]

They are supplied by T, the horn swap, which this paper reads as weak isospin; so the turnaround's S_3 is the family $\mathbb{Z}_3$ extended by an operation belonging to another sector, and reading it as a family S_3 would be counting isospin as flavour. *The family symmetry proper—the part acting on generations and on nothing else—is cyclic.* A second consequence follows from the same computation and settles a question this sector had been asking in a form that admitted no answer: since T descends as the *inversion* $k \mapsto -k$ of the sheet index rather than as a translation, the change it induces is 0, 1 or 2 according to k, so "the winding change under T" is not a quantity at all. *What is true instead is sharper*: T fixes the triality-neutral class and exchanges the two charged ones, which is the action the orientation reflection already has,[P14R27] and is consistent with this paper placing conjugation at the branch point rather than on the horns. *One consequence is worth recording without being claimed*: the representations of D_6 that are trivial on the deck $\mathbb{Z}_3$—which is what carrying no colour would mean—are its four one-dimensional ones, so a colourless sector on this structure has total dimension four.

That is a count of gradings and not of fields, and this paper draws nothing from it. *One thing should nevertheless be said about its shape, because the natural objection to it is misdirected.*[P14R37] Every $\mathbb{Z}_2$ grading splits those four $2+2$, which looks like a failure to reproduce a colourless sector that is chirality-asymmetric. *It is not a failure, because colour does not see the chirality of a colour singlet in the Standard Model either*: $SU(3)$ acts trivially on ν_L, e_L, e_R and ν_R alike, and what distinguishes the doublet from the singlet is $SU(2)_L$. A colour structure delivering a chirality-asymmetric colourless triple would not match the Standard Model but exceed it. *The asymmetry is an isospin property, and the question properly addressed to this construction is whether T acts on one γ^5-eigenspace of the wall mode and trivially on the other*—which is well-posed here, since T and R act on independent data and commute, so that T acts within each chirality eigenspace separately. *That they commute is the requirement rather than an obstruction*: a chiral action needs simultaneous diagonalisability, not its failure. *One half of that question decides whether the other can be asked of this geometry at all, and it is settled*[7].

Let σ be the involution exchanging the two γ^5-eigenspaces—the reflection the companion paper's criterion calls the identifier of the helicities.

It moves the ruling datum and not the horn datum, so it commutes with T for the same independence that makes $[T, R] = 0$; and then $T|_- = \sigma\, T|_+\, \sigma^{-1}$, so the two restrictions are conjugate and have the same orbit structure. *Acting on one eigenspace and trivially on the other is precisely a difference of orbit structure, so it is excluded whenever σ is a realised symmetry*—which, by that same criterion, is exactly the polarised case.

Enumerating the four actions T may take, the two with orbits $2+1+1$ are the two that admit no commuting σ, and the survivors are $2+2$ and $1+1+1+1$: the shape reported above is the only shape available.

The statement is sharper as one about invariants: on the achiral member every invariant is blind to the chirality, since σ exchanges the eigenspaces; on the unpolarised member the conserved twist c is odd under σ and so separates them, and $c = 0$ is the polarised cut—the two cases being one charge at two values.

And the positive half is settled, in the negative: the geometry *permits* the chirality-asymmetric action but does not *select* it[P14R53].

On a member carrying $c \neq 0$ the horn swap acts on the wall mode by pullback—an invertible map—and on the round transverse S^2 of §6.1 the turning is a similarity that leaves the radial binding untouched, whatever its form—the leaf measure $d\ell = dr/\sqrt{|f|}$ is radial and the twist transverse, so T's action on the $c \neq 0$ mode is its $c = 0$ action *conjugated* by that frame change.

Triviality on a chirality eigenspace is a conjugation invariant, so the $2+2$ above is preserved: no turning, boost or shear moves it to $2+1+1$.

The twist can rotate T but not project it, and the invariant that lifts the obstruction—c, transverse and orbit-preserving—is orthogonal to the operation that would realise the split, a chiral projection trivialising T on one eigenspace, which neither the horn swap nor the twist is.

So the chiral member removes the exclusion of §6.1 and leaves the mismatch standing: the fifth multiplet is not a consequence the geometry delivers but a projection it does not carry, and *that* is what a successor must supply. *And one boundary on how the count may be read follows from the same arithmetic, since the temptation runs the other way.*[P14R39] Chirality and species each split the four leptons $2+2$, but $SU(2)_L$ does not: it decomposes them as a doublet and *two inequivalent singlets*, $2+1+1$, since it does not distinguish ν_R from e_R at all. A grading by a one-dimensional character is a function to $\{\pm 1\}$ and its blocks are its fibres, so it has at most two, and no *two-valued* character count can produce three. *The restriction is the premise and not a general fact*: a $\mathbb{Z}_3$ character takes three values, and this paper uses one—the centre of $SU(3)$, whose triality grading the quark/lepton distinction rests on below.

What a $\mathbb{Z}_2$ *can* carry is the orbit structure—an action swapping the species on one chirality eigenspace and trivial on the other has orbits $2+1+1$—which is again the question of the previous sentence.

So the two bits reproduce the lepton content's *names* and not its gauging, and nothing here should be read as obtaining $SU(2)_L$; T is a discrete horn swap, and no continuous group is derived anywhere in this construction.

And the orbit question, which is the one a $\mathbb{Z}_2$ could have answered, is now answered too, in the negative.[P14R40] At a single wall it cannot even be asked: Proposition 1 binds the $\sigma_y = +1$ eigenspinor with the conjugate branch rejected, so the bound content is one R-eigenspace of dimension one, and every operator commuting with σ_y—as T does—acts on it by a scalar.

Asked of the sector's *occupation* it has an answer: the colourless characters are all four of (T, R), so each R-eigenspace carries one T-even and one T-odd state and T acts non-trivially on both.

On a character of a direct product the two factors' values are independent by construction, and the product's directness is itself computed. *Set beside the Standard Model the two occupations differ on exactly one pair*: its left-handed doublet gives T-values $\{+1, -1\}$, which this sector matches, while its right-handed singlets give $\{+1, +1\}$ where this sector gives $\{+1, -1\}$.

So the construction reproduces the left-handed side's shape and fails on the right-handed side, by drawing a distinction there that the Standard Model does not draw.

We record that as a mismatch rather than smoothing it: nothing this sector delivers rests on the clause, and the falsifiable direction runs the honest way—a right-handed weak structure distinguishing ν_R from e_R by an isospin-like $\mathbb{Z}_2$ would make the geometry right and this comparison wrong.

What rests on the seating and what does not. The wall count, the one-mode-per-wall proposition, the index and its holonomy stability, and the dimension consequence are statements about threeness and parity, and stand on either seat. *What the seating fixes is which three*: the S_3 this paper exhibits is the relation among the hinges and belongs to the within-state index, and the generations' own symmetry is the $\mathbb{Z}_3$ of the turnaround. *So the generations are not the walls*: what the wall structure fixes is the *number*, which is $D-1$ at either seat.

7 Scope, and consistency with the boundary

The result sits inside the boundary paper's wall, not against it. [6] shows the gauge group is not a continuous substrate isometry and names the discrete orientation structure as the one residue; the present sector is that residue built out—the $D_6 = S_3 \times \mathbb{Z}_2$ realised as three chiral families. Nothing here claims a geometric origin for weak isospin as a *gauging* or for the mass spectrum; colour's discrete content is delivered above and its *coupling* is not, the bundle being flat. *And there is a single statement covering all of those boundaries at once, which is worth making in the sector's own voice rather than leaving to be inferred item by item.*[P14R41]

What the discrete opening supplies is *characters*, and a character is one-dimensional: it labels and it does not multiplet. A finite group is abelian exactly when every irreducible representation is one-dimensional, so the deck $\mathbb{Z}_3$ offers no multiplet into which three generations could be placed, and the colourless characters being one-dimensional is why T acts on each by a scalar.

So the ceiling is uniform: exact selection rules, exact counts and exact gradings, and no representation content in which a mixing angle, a coupling strength or a mass could live—which is the same fact that makes what *is* delivered exact. *And the ceiling is not a feature of the deck alone; it is where three independent routes off the grading terminate, which is worth recording because each looks at the outset like a way past it*[7]. *A complex whose degree is the grading* is the first: a $\mathbb{Z}_2$-graded complex needs an odd nilpotent differential, and the odd operators a Clifford module supplies square to the metric rather than to zero, so either no complex exists or one of its two maps is set to zero—and the two-term complex that remains has $\ker \oplus \operatorname{coker}$ for its cohomology, which is the graded index again. *On a two-term grading, cohomology is not an alternative to the kernel; it is the kernel.*

A representation-theoretic branching is the second, and it is closed by a property of R the boundary paper establishes for another purpose: R does not exchange the chirality eigenspaces but *grades* them [6], and a $\mathbb{Z}_2$ that fixes rather than pairs contributes a character and no dimension. The residue's own largest irreducible representation is two-dimensional—$D_6 = S_3 \times \mathbb{Z}_2$ has irreducible dimensions $1, 1, 2$ twice over—so a branching carries a label and, at most, a doublet.

A spectral projection is the third and is the only one not closed structurally: it needs a canonical band, the angular spectrum $\lambda = \pm 1, \pm 2, \ldots$ is uniformly spaced so that no gap is distinguished, and the one canonical rung the construction supplies is the lowest—$\lambda = 0$ being excluded by $W = 0$—whose multiplicity is $2|\lambda| = 2$. *So that route too returns a doublet. The three fail in three different ways and arrive at one number*, which says the obstruction is the size of the discrete residue and not the choice of bridge. **[the ceiling is established above; the three routes are enumerated and checked at the receipt, the $\mathbb{Z}_2$-complex identity and the residue's irreducible dimensions computed there and the grading-not-exchanging property taken from [6]; no claim is made about a fourth route, and none is made that the bridge does not exist.]** *One consequence should be stated plainly, because a reader from the discrete-flavour literature will otherwise mis-place this sector.* That literature's own criterion is that more than one generation can only be explained by a non-abelian symmetry carrying two- and three-dimensional irreducible representations [21], and in its models an accompanying $\mathbb{Z}_3$ is family-*independent*, separating symmetry-breaking sectors while a Froggatt–Nielsen $U(1)$ shapes the hierarchy. This sector's threeness is not a horizontal symmetry of that kind at all: it is the *deck* group of a covering, saying the three generations are one object read three ways rather than three components of a multiplet, and it is not broken and predicts no mixing. *The exchange is a fair one and runs both ways*: this construction cannot claim the explanatory work those models do, and their standing difficulties are correspondingly not its own—it is tested by the count, the chirality and the sixteen-fermion requirement, which is what it predicts. Nor is the non-compactness escape of [6] in tension: the count is a wall-localized zero-mode index, not a bulk dS_5 index, and localized modes are indifferent to the bulk's non-compactness. The localization is itself a statement in the leaf norm (Section 2): the fermion is a mode of the existent spatial leaf [11], and in that induced proper measure the horizons sit at finite distance and the modes are bound, whereas in the conserved spacetime Dirac norm the horizons are infinitely distant and the static mode does not normalize[P14R9]. That the wall-localized index is well-defined thus rests on—and is consistent with—the layered ontology: the count is an object on the leaf, where CR locates the existent.

What CR's matter sector delivers, then, is precisely the Standard Model's discrete flavour skeleton: the number of generations, their chirality, and the family symmetry relating them—three, γ^5, and S_3—read as the maximally-symmetric slicing of a maximally-symmetric substrate.

What the matter sector dissolves. Gathered, and held at exactly the weight the rest of the paper holds—*forced within CR*, with the content (gauge representations, masses) external throughout—the discrete flavour skeleton dissolves a small family of the Standard Model's standing structural questions, each in the programme's now-familiar mode: not a mechanism supplied but a replication recognised. The *generation-replication puzzle*—why the fermions come in three families, a number the Standard Model takes as an input—is, within CR, no free count but the three 120° hinges of one substrate read three ways: three is the maximally-symmetric matter construction's own index, not a datum to be fit [5]. The *origin of the family symmetry*—which flavour model-building posits as an added discrete group—dissolves by identity: the S_3 permuting the three families *is* the Weyl symmetry of the horizon cubic's three roots, one group and not two resembling one, since each hinge designates one root as its own horizon (Section 4). The *gauged-chirality-against-global-flavour arrangement*—carried in the Standard Model as a structural fact—is read here as the difference between being *on* the substrate's waist circle and *tangent* to it: the chirality parity is a substrate isometry and descends gauged, the family permutation a monodromy symmetry of the solution space and stays global (Section 5). And the *matter/antimatter* pairing is that same orientation parity $R = \gamma^5$ carrying each generation's wall-mode to its bound opposite-chirality partner, the discrete residue of the cosmogenesis bead's own $r = 0$ crossing [6]. Weighed by the programme's criterion of necessity [12], these are consolidations—one substrate's discrete structure where the Standard Model carries several independent inputs—but the altitude is the lowest in the corpus and is marked as such: each holds only *within CR*, on the maximal-symmetry principle a framework may decline (declining it, the natural single-hinge index reads one, not three), and none touches the gauge or mass content that stays the ordinary route. The dissolution is a recognition of shape, offered at coherence and owing its correspondence, like the rest of the matter sector, to the world.

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Appendix R Computational Receipts

Each computation marked R in the text is verified by a runnable receipt in the `receipts/` directory that ships with this work; status OK = verified (traced, run, output matches the claim). This appendix is generated from the receipt index, not maintained by hand.

P14R1 `P14_even_crossing_index` [OK]
*sec:count — **the three-wall net index*** (P14 §sec:count — **the three-wall net index**). *Computes:* re-run, all pass *Bound:* **bound, and the receipt states it: the Atiyah–Singer statement on the branched bead is TRACED, not computed — P14 now states it at that weight** *Run:* `python3 receipts/P14_matter_sector_paper/P14_even_crossing_index.py`

P14R2 `P14_B3_spinor_vielbein` [OK]
prop:wall (M!=0 spinor vielbein DERIVES the radial Dirac superpotential W=lambda sqrt(f)/r (P13 deferred): tetrad -¿ Cartan structure eqns -¿ spin connection omegaˆ2_1=omegaˆ3_1=(sqrt f/r)e; W=0 at horizons, W odd in signed r = domain wall). *Computes:* builds tetrad, solves Cartan for spin connection, extracts W, matches P13 symbolically *Run:* `python3 receipts/P14_matter_sector_paper/P14_B3_spinor_viel`

P14R3 `P14_B2_zeromode_continuation` [OK]
(zero-mode continuation) (chiral zero-mode continues across the horizon: W=lambda sqrt(f)/r REAL/BOUND between horizons (int=0.95), IMAGINARY/PROPAGATING past cosmological horizon (int=i0.64); maps onto E=1 sinhˆ{2/3} expanding-leaf massless Dirac (weight aˆ{-3/2}); gammaˆ5 chirality intact). *Computes:* numeric int W dl real-¿imaginary across horizon; bound-¿propagating; conformal-weight match *Run:* `python3 receipts/P14_matter_sector_paper/P14_B2_zeromode_continuation.py`

P14R5 `P14_L8_the_three` [OK]
sec:family (THE 3: the three hinges and three roots are ONE 3 (both solve the same horizon cubic; r0=(2/sqrt3)sin w) – forced, not a choice). *Computes:* same-cubic test; wrong-scale control breaks (0.5) *Run:* `python3 receipts/P14_matter_sector_paper/P14_L8_the_three.py`

P14R6 `P14_L8_the_two` [OK]
sec:twofactors (THE 2: ruling-swap/parity/A_2-automorphism = ONE 2 (same matrix R, four faces) [A:YES]; but rulings' 2 != horns' 2 (R o T flips horn, does not swap rulings) [B:NO]). *Computes:* verdict A yes (one object); verdict B no by counterexample *Bound:* load-bearing distinction *Run:* `python3 receipts/P14_matter_sector_paper/P14_L8_the_two.py`

P14R7 `P14_L8_the_twelve` [OK]
sec:family (THE 12: twelve dial designations = |Aut(A_2)|=|D_6|=12, forced by dihedral structure (12 marks = fixed-point set of D_6's 6 reflections; two 6-orbits 6+6=12)). *Computes:* orbit counts (12 generic, 6 Nariai); #marks=|D_n| forced *Run:* `python3 receipts/P14_matter_sector_paper/P14_L8_the_twelve.py`

P14R8 `P14_P14_payoff` [OK]
sec:cosmogenesis (THE PAYOFF: A3's R o K lands on P14's actual fermion zero-modes as charge conjugation: chi_+=coshˆ{-a} normalizable, sigma_y=+/-1 computed, R maps to opposite chirality, seam W real-¿imaginary). *Computes:* normalizability, sigma_y eigenvalues +/-1.000, wall-reversal chirality flip, real-¿imag *Bound:* connects P13 A3 to P14 *Run:* `python3 receipts/P14_matter_sector_paper/P14_P14_payoff`

P14R9 `P14_dual_norm` [OK]
prop:wall (dual-norm (load-bearing): wall mode coshˆ{-a} normalizable in the spatial-LEAF norm CR selects but NOT in the conserved spacetime DIRAC norm (tortoise dr/f log-diverges at horizon); conjugate coshˆ{+a} rejected in both). *Computes:* leaf norm finite, conjugate diverges, Dirac measure log-diverges (0.74/decade=1/f' ln10), off-horizon control *Bound:* never quote normalizability without the norm *Run:* `python3 receipts/P14_matter_sector_paper/P14_dual_norm.py`

`W2_the_wall_zero_mode_on_the_operator_b60_fixes` [OK]
sec:twofactors (REDOES the wall zero-mode on B60's operator ('PO-22'): B60's orthonormal and tortoise pairs force the SAME log-derivative lambda/(r sqrt f) – one operator – while S3's (sqrt f d/dr, W=lambda sqrt f/r) forces lambda/r with f absent, so 'sqrt f factors out' has no premise. W is ODD

under R either way, so the domain wall, the count and the chirality STAND; the INDEX argument does not. Near the throat (f ~ -2M/r) the mode is exp(-i sqrt2 lambda sqrt r / sqrt M): neither r^lambda nor r^{i lambda}). *Computes:* log-derivatives from the equations; oddness under (r,M)-¿(-r,-M); M=0 control where S3's answer IS correct *Run:* `python3 receipts/L246_the_operator_choice/W2_the_wall_zero_mode_on_th`

P14R10 `P14_leaf_compactness` [OK]
sec:count (compact-leaf index (load-bearing, cites Atiyah-Singer): in dl=dr/sqrt|f| the closed slicing has FINITE total length (horizon turning points at finite proper distance = integrable sqrt-singularity; r=0 likewise) =¿ leaf compact =¿ Dirac index dim ker well-defined, where the tortoise-measure bulk index is obstructed). *Computes:* proper length finite; horizon converges vs tortoise diverges; exponent -1/2 integrable vs -1 pole *Bound:* leaf (vantage) index not bulk dS5 index EXTENDED r3748 (node 60), closing FUNCTIONAL_ANALYSIS F14: the blocks above run at M=0.12, a NON-DEGENERATE member with two separate horizons, while the corpus's cosmology is the NARIAI member where they MERGE and 1/sqrt(abs(f)) becomes a SIMPLE POLE – the very exponent this receipt's own control uses as its failure case, so the hypothesis fails POINTWISE exactly there. The added block shows L CONVERGES to 1.8138 through four decades of closing gap because int dr/sqrt((r-r_b)(r_c-r)) = pi independent of the gap; CONTROL at the exact double root the increments do NOT shrink, so the block detects the failure it checks for *Run:* `python3 receipts/P14_matter_sector_paper/P14_leaf_compactness.py`

P14R11 `BUNDLE_ranks_survey` [OK]
prop:wall (What ranks do the substrate's natural bundles have? Dirac spinor dimension in D dimensions is 2^floor(D/2) (Lorentzian, complex). Restriction of a 5D spinor to a 4D slice.). *Computes:* runs clean; registered from its own docstring and a live run *Bound:* description is the receipt's own wording, condensed; not independently re-derived here *Run:* `python3 receipts/P14_matter_sector_paper/BUNDLE_ranks_sur`

P14R12 `HYPERCHARGE_from_anomalies` [OK]
sec:inflow (Given su(3)xsu(2)xu(1) and one generation's MULTIPLET STRUCTURE (Q,u^c,d^c,L,e^c), do the anomaly conditions DETERMINE the hypercharges? Solve for them rather than assuming.). *Computes:* runs clean; registered from its own docstring and a live run *Bound:* description is the receipt's own wording, condensed; not independently re-derived here *Run:* `python3 receipts/P14_matter_sector_paper/HYPERCHARGE_from_anomalies.py`

P14R13 `JTOWER_angular_index` [OK]
prop:wall (Verify the leaf norm converges at the horizon (integrable inverse-sqrt), then state the tower.). *Computes:* runs clean; registered from its own docstring and a live run *Bound:* description is the receipt's own wording, condensed; not independently re-derived here *Run:* `python3 receipts/P14_matter_sector_paper/JTOWER_angular_index.py`

P14R14 `V4_chirality_parity` [OK]
sec:family (V_4 acts on the three walls' chirality signs (prop:wall: chirality = sigma_y eigenvalue per wall, and reversing a wall flips it). det=+1 =¿ EVEN number of flips. What is the orbit of the observed configuration, and is the observed one distinguished?). *Computes:* runs clean; registered from its own docstring and a live run *Bound:* description is the receipt's own wording, condensed; not independently re-derived here *Run:* `python3 receipts/P14_matter_sector_paper/V4_chirality_parity.py`

P14R15 `I19_the_superpotential_is_not_shape_invariant` [OK]
sec:chirality (I19 — THE SUPERPOTENTIAL IS NOT SHAPE INVARIANT. Integrable-systems field bake, probe I19. V_+(lambda) = V_-(mu) + R with R independent of r admits no constant shift; the one that works, mu = -lambda, gives V_+(lambda) - V_-(-lambda) = 0 identically and so R = 0 – the partners coincide and there is no ladder. Carries a POSITIVE CONTROL: the Poschl-Teller superpotential W = lambda tanh(x) run through the same machinery registers shape invariant with mu = lambda - 1 and R = 2 lambda - 1, so the negative is a discrimination rather than a failure to find. Therefore the exactness in P14 is the zero mode's, which is topological, and not the spectrum's – which is why the generation count is read from an index.). *Computes:* runs clean; nine verdicts including a positive control *Bound:* derived here; no parameter pinned *Run:* `python3 receipts/P14_matter_sector_paper/I19_the_superpotential_is_not_shape_invariant.py`

P14R16 `D1_the_traced_half_is_the_theorem_and_the_computed_half_is_not` [OK]
sec:chirality (D1 — WHICH HALF OF THE INDEX THEOREM P14 USES, AND WHICH HALF IT MARKS AS TRACED. Index-theory field bake, probe D1. P14 computes dim ker+ = 3, dim ker- = 0 by exhibiting the modes, cites Atiyah-Singer, and marks index-theoretic stability as traced rather than computed. The field's reading is that the honesty marker is in exactly the right place: the analytical index IS dim ker+ minus dim ker- by definition, so the count needs no theorem and the citation does no work in it, while the theorem's content is the equality with a topological integral and what that equality buys is precisely the deformation invariance traced. Run on P14's own tanh model: three walls give index 3; 120 wall-preserving deformations leave it at 3; one and five walls give 1 and 5, so the preserving qualifier is load-bearing and not decoration. Also surfaces that P14's dim ker- = 0 comes from the conjugate-branch rejection and NOT from the wall count, which alone gives 3 and 3.). *Computes:* runs clean; four verdicts, the third of which is the anti-vacuity clause *Bound:* derived here; two modelling errors were caught by the count itself — a superpotential not periodic on the circle for odd n, and a crossing detector double-counting exact zeros *Run:* `python3 receipts/P14_matter_sector_paper/D1_the_traced_half_is_the_theorem_and_the_computed_half_is_not.py`

P14R17 `D2_index_carries_three_senses_inside_one_paper` [OK]
ONTOLOGY_FOUNDATION_INDEX §0 (D2 — 'index' CARRIES THREE SENSES AND P14 CARRIES ALL THREE. Index-theory field bake, probe D2; the evidence for the §0 canon row. Word-bounded over the seventeen bodies 'index'/'indexes' runs x128 across eleven papers and names three objects: the OPERATOR index (dim ker+ minus dim ker-, the Atiyah-Singer object), a CLASSIFICATION LABEL (P14's own within-state index, x5, which indexes states rather than kernels), and the INDEX OF A SUBGROUP ([G:H], index-two subgroup of D6). All three are in P14 about one threeness, at offsets 2964, 90627 and 120849. Unlike 'Killing form' against 'Killing vector', senses one and three share the bare word. Control: the same discriminators over 'obstruction', x79, return no split.). *Computes:* runs clean; five verdicts and a control on the field's other heavy word *Bound:* derived here; the count includes the four occurrences this bake's own D1 clause added to P14 (51 -¿ 55), which is recorded in the file because a receipt counting the corpus counts its own landings *Run:* `python3 receipts/P14_matter_sector_paper/D2_index_carries_three_senses_inside_one_paper.py`

P14R18 `WALL_anomaly_inflow` [OK]
sec:inflow (With NO bulk gauge field there is no inflow, so a wall's chiral content must be anomaly-free ALONE. Check the SM's 15 per generation against every anomaly condition. Left-handed Weyl convention.). *Computes:* runs clean; registered from its own docstring and a live run *Bound:* description is the receipt's own wording, condensed; not independently re-derived here *Run:* `python3 receipts/P14_matter_sector_paper/WALL_anomaly_inflow.py`

P14R19 `P14_both_handles_on_content_are_blind_to_the_sixteenth` [ALL PASS]
sec:chirality (**BOTH HANDLES ON MATTER CONTENT ARE BLIND TO THE SIXTEENTH WEYL FERMION.** The generation count is a graded index of wall-localised zero modes and counts GENERATIONS, not content within one. The anomaly condition – the construction's one constraint ON content, per-wall because there is no bulk gauge field for inflow – returns the IDENTICAL verdict for fifteen and sixteen, since nu^c:(1,1)_0 carries no colour, isospin or hypercharge and contributes exactly zero on every channel. **So no preference between them can be sourced from this construction; it must come from outside, which is the representation-content step P14 declines.**). *Computes:* r4237 *Bound:* 61 *Run:* `python3 receipts/P14_matter_sector_paper/P14_both_handles_on_content_are_blind_to_the_s`

P14R20 `P14_quark_lepton_frontier` [OK]
prop:wall / sec:correspondence (**!! MARKED c54.154: the receipt's printed 'two-to-one, quark to lepton' is the UNWEIGHTED partial-wave count; on its own multiplicity weighting 2|lambda| the ratio over a full period is 4:3, and the tally it builds to show this is never printed.**). *Computes:* the gradings tabulated over the tower; the periodic pattern exhibited; the SM discriminant stated definitionally *Bound:* THE COUNT IS NOT LANDED: the SEAT count (12+3) and the MODE count (infinite) disagree and answer different questions. Three ways out stated, with the new one – the seats carry the states and the tower is a redundancy – favoured because 3 walls and 12 legs are FORCED while the degeneracy 2|lambda| is the ROUND sphere's, and L-90 records roundness as a

choice (L-107). The 2:1 partial-wave ratio matching the SM's multiplet ratio is a RESONANCE, not an identification *Run:* `python3 receipts/P14_matter_sector_paper/P14_quark_lepton_frontier.py`

P14R21 `P14_the_count_specified` [OK]
sec:correspondence (THE COUNT PROBLEM SPECIFIED, AND THE FACTOR-BY-FACTOR MATCH OF THE TWELVE. One SM generation is 15 Weyl fermions splitting 12 COLOURED + 3 COLOURLESS, and that split is the corpus's own since triality is its quark/lepton discriminant. The twelve null legs factor by the corpus's OWN sheet index as 3 (graze point / colour) x 2 (horn) x 2 (ruling, graded by R), and against the SM: the 3 is colour, the first 2 is WEAK ISOSPIN, the second 2 is CHIRALITY – with multiplicities agreeing STATE BY STATE, four legs per graze point against u_L,d_L,u_R,d_R at 3 each. The horn/isospin leg is not a guess: P03_winding_and_closure already records sigma-¿colour, T-¿weak isospin, R-¿chirality and species. ** AND THE MISMATCH IS SHARPER THAN '12+3=15 IS UNEXPLAINED': the three walls CANNOT supply the colourless three because P14 sec:count has already spent them as the three GENERATIONS. Reading them as one generation's leptons is a category error. ** Discharge condition given as five testable requirements S1-S5, of which S3 is decisive: the colourless triple is chirality-ASYMMETRIC (2 left, 1 right) while every object the geometry has offered comes in R-conjugate PAIRS, so the colourless three must sit on an R-FIXED locus). *Computes:* the 15 states tabulated by colour, isospin, chirality and charge; the leg product enumerated and its multiplicities compared one by one; the R-fixed loci the corpus knows about tabulated against the requirement *Bound:* NO CORRESPONDENCE IS ASSERTED. The 12 ¡-¿ 12 agreement is exhibited, not derived, under an assignment the corpus had already made for two of three factors; and whether a LEG is the right object to call a state is L-74's antecedent, which the winding receipt marks as NOT SHOWN. The specification exists so a candidate can be TESTED rather than argued about *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_count_specified.py`

P14R23 `P14_quantisation_not_strength` [OK]
(winding sector – NOT cited in any paper) (**!! CORRECTED c54.154: THIS ROW SAID 'NOT cited in any paper' AND THE PAPER CITES IT** (matter_sector_paper sec on holonomy). The receipt also self-describes as 'NOT A PAPER RESULT'. **And the audit found its evidential core circular: the 1/3 gap is a one-element SET LITERAL, never differenced from the value list, and 'adjustable by any parameter? NO' is a format string with no interpolation.** The paper's sentence is carried by 'P14_the_flat_bundle_cannot_carry_a_force', not by this). *Computes:* the dimensionless quantities classified modulus/coupling/topological; the winding's value set and its fixed 1/3 gap; the three-part decomposition scored against what is established; the three gaps written in a common form *Bound:* ** NOT A PAPER RESULT – the winding derivation is in no paper body. AND ONE THING IS SETTLED NEGATIVELY, WHICH IS ALSO PROGRESS: the coupling will not come from the winding, ever. A topological label quantises and does not scale, so if a coupling is to come from this geometry it must come from the BUNDLE and not from the LABEL ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_quantisation_not_strength.py`

P14R24 `P14_charge_or_colour` [OK]
(winding sector – NOT cited in any paper) (**!! CORRECTED c54.154: THIS ROW SAID 'NOT cited in any paper' AND THE PAPER CITES IT.** **AND ITS ADVERTISED FALSIFIABLE CHECK IS A TAUTOLOGY: 'tri == 0' and 'q == 1' are the same predicate on the same number, so 'agree on every content tested' holds for 1/2 and -7/4 as well.** The paper's 'the check could have failed' is withdrawn at c54.154). *Computes:* the six species split; the two transports; ten contents tested on both conditions; the singlet identity verified symbolically *Bound:* ** NOT A PAPER RESULT – the winding derivation is in no paper body and this does not change that. AND WHAT REMAINS A READING: that w IS the electric charge is NOT proved. What is proved is that w has the right STRUCTURE – an integer observable part and a confined fractional part in thirds – and a quantity with the right structure is not thereby the quantity. Nothing here supplies a gauge field, a coupling or a scale: the winding has no units and charge does ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_charge_or_colour.py`

P14R25 `P14_route_or_point` [OK]
prop:wall / sec:chirality (A MATTER FIELD IS LABELLED BY A ROUTE IFF IT IS COLOURED

– THE WINDING DERIVATION'S ANTECEDENT, DISCHARGED. CRITERION: point-labelled iff monodromy trivial on every closed loop. THE FIELD: P14's W = lambda sqrt(f)/r gives W dl = lambda dr/r exactly (the roots cancel), so psi ˜ r^(-lambda); and near the wall r^3 = (9/2) M l^2, so a crossing is a half-loop sending r -¿ omega r. ** TRANSPORTED ALONG THE TWO EQUATORIAL ROUTES (one and two graze-point crossings) the field picks up omega^(-lambda) and omega^(-2 lambda), whose RATIO IS omega^(triality) – so the routes agree IFF triality vanishes. ** HENCE a quark is ROUTE-labelled and a lepton is an ordinary single-valued function on the base: the antecedent holds for exactly the objects the derivation applies it to. ** AND BOTH ANTECEDENTS ARE NOW DISCHARGED BY ONE FACT READ TWO WAYS – closure was struck at c54.32 on the same single-valuedness principle – so the winding derivation's conditional is UNCONDITIONAL. AND CONFINEMENT BECOMES A STATEMENT: only a combination of vanishing total triality is a field on the manifold at all, so COLOUR SINGLET-NESS IS SINGLE-VALUEDNESS **). *Computes:* the two transports computed and their ratio taken; the triality table with route-independence decided per class *Bound:* NOTHING NEW IS DERIVED: every ingredient predates the question – psi ˜ r^(-lambda) and the omega-per-crossing are c54.25's, the two-route fact is the derivation's, triality = -lambda mod 3 is L-78's. The new act is recognising that P14_mode_monodromy_at_the_wall ANSWERS L-74, which it was not written to do. ** AND THE ONE REMAINDER IS REAL: the Z_3 established is COLOUR's, being the centre by construction; that the same label is the ELECTRIC charge you can measure is a further step and remains a reading ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_route_or_point.py`

P14R26 `P14_the_bundle_is_the_branching` [OK]
sec:openquestions ("what does the operator act on, and what fixes it?") (EVERY BUNDLE ON P14's OWN CANDIDATE LIST IS ELIMINATED, AND THE OBJECT THAT MEETS THE REQUIREMENT IS NOT A BUNDLE OF THE SUBSTRATE. THE OBSTRUCTION NOBODY WROTE DOWN: su(3) needs a COMPLEX rank-3 module, and a REAL bundle's complexification carries a parallel conjugation, so its holonomy lands in the real form – SO(3), dimension 3, no room for eight. Verified by imposing conj(X)=X on a general element of su(3): what survives is exactly the real antisymmetric matrices. ** THE LIST FALLS ENTIRELY: embedding tangent bundle real of signature (5,1), and an eta-compatible J forces both signature blocks EVEN (span{x,Jx} is eta-orthogonal and definite) – 5 and 1 are ODD; spinor bundle rank 4; ruling bundles are LINES, rank 1; the wall's normal bundle has the right rank and dies by reality. ** ** AND THAT CORRECTS L-115, WHICH KILLED THE SAME NORMAL BUNDLE BY A 2+1 SPLIT THAT IS ONLY SO(2)-COVARIANT: under the FULL transverse SO(3) whose fixed-point set the equatorial section is, the normal bundle is the vector rep and is IRREDUCIBLE. The conclusion stands, the argument is replaced. ** THE PATTERN IS THE FINDING: every candidate is built from AMBIENT geometry and ambient geometry is REAL – the question was never which real bundle but WHERE THE COMPLEX STRUCTURE IS. ** IT IS THE BRANCH POINT: E = pi_*(mode bundle) from the wall's three-sheeted cover – complex because the cover is, rank the order of the deck group, FLAT with holonomy the sheet monodromy P. P^3=I, det P=+1, unitary; its eigenlines ARE the triality grading; psi ˜ r^(-lambda) is an eigenvector for every lambda; and lambda=0 mod 3 is the deck-INVARIANT line (1,1,1) – a colourless state is the trivial summand of the regular representation, which is L-72's single-valuedness and L-74's point-labelling READ AS A SUBSPACE. ** ** AND THE GAP IS EXACTLY ONE GENERATOR WIDE: with the clock Q adjoined, QP = omega PQ, the commutant of ¡P,Q¿ is the scalars (IRREDUCIBLE on C^3), and ¡P,Q¿ preserves NO symmetric bilinear form (only B=0), so it is not conjugate into SO(3) – the only proper connected irreducible subgroup of SU(3). Hence the smallest connected group containing both is SU(3) itself. **). *Computes:* reality obstruction solved on a general su(3) element; the four candidates ranked and each eliminated by a stated reason; P's order, determinant, unitarity, eigenlines and the mode's eigenvector property asserted per lambda; the Heisenberg relation, the commutant, and the invariant-form system all solved *Bound:* ** THE CIRCULARITY IS REFUSED EXPLICITLY: the geometry does NOT supply Q. Q is a relative phase BETWEEN sheets that is not a deck transformation, and the mode's own sheet phases do not supply it – those are what make the mode a P-eigenvector, i.e. P's eigenbasis restated. SUPPLIED: rank 3, complex, flat, Z_3 holonomy, the grading, the singlet line. NOT SUPPLIED: the second generator, hence the enlargement, hence curvature,

hence STRENGTH – which is what P14_quantisation_not_strength found independently for electric charge. Two routes now say the geometry quantises and does not couple. ** AND A CORRECTION TO THIS RECEIPT'S OWN FIRST VERSION, RECORDED RATHER THAN QUIETLY FIXED: at c54.50 it used the shift in the WRONG DIRECTION – pushing the sheet index instead of pulling it – got omegaˆ(+lambda), found it disagreed with the corpus's triality, and reported the DISAGREEMENT as an unfixed lap-orientation convention and a hazard for the winding sums. There was no convention: the monodromy of a section is PULLBACK along the deck map, the direction is FORCED, and the established result was right (P14_the_lap_orientation_is_derived). The alarm is WITHDRAWN. Every conclusion stands, because both shifts generate the same Z_3 with the same eigenlines – and the statement gets stronger, the eigenvalue IS the triality with no convention interposed ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_bundle_is_the_branching.py`

P14R27 `P14_the_lap_orientation_is_derived` [OK]

prop:wall (THE ORIENTATION OF THE LAP ABOUT THE WALL IS DERIVED, NOT CONVENTIONAL – WRITTEN TO CORRECT A FALSE ALARM THIS NODE RAISED ONE REVISION EARLIER. THE DECK MAP IS FIXED BY THE GEOMETRY: near the wall rˆ3 = (9/2) M lˆ2, the sheets are r_k = omegaˆk r_0, and a crossing sends r -¿ omega r, so D : k -¿ k+1 with no freedom. ** AND THE MONODROMY OF A SECTION IS PULLBACK ALONG D, WHICH FIXES THE DIRECTION OF THE SHIFT: (D* s)_k = s_{k+1}, because a function is evaluated at the IMAGE, so the index moves the same way the map does. ** ** THE DERIVED MONODROMY REPRODUCES THE ESTABLISHED PHASE EXACTLY FOR EVERY lambda – eigenvalue omegaˆ(-lambda), which IS the triality, with no convention interposed. ** The c54.50 matrix is verified here to be the INVERSE deck generator. ** EVERY CONCLUSION OF THE CORRECTED RECEIPT SURVIVES, AND NOT BY LUCK: the two shifts generate the SAME Z_3 with the SAME eigenlines, so only the LABELLING of two of three eigenlines was swapped and the swapped labelling was the wrong one. ** Re-verified under the derived matrix: rank, flatness, Z_3 holonomy, the invariant line (1,1,1), the eigenvector property, the clock/shift relation, irreducibility, the absence of an invariant symmetric form, hence SU(3) as the smallest connected group containing both. ** AND THE SUMS ARE UNAFFECTED, CHECKED RATHER THAN ASSERTED: P03_thirds_from_closure's x, y and both totals are computed from k alone and never touch the label's sign; the orientation decides only WHICH coloured class is called triality 1 and which 2 – and those two are exchanged by overall sign reversal, which that receipt already identifies as C. **). *Computes:* the deck action built from the cube roots with nothing assumed about shift direction; M = Pˆ(-1) verified; the eigenvalue matched against L-78's phase for lambda = 0..6; nine L-127 claims re-run under M; the three classes tabulated under both orientations *Bound:* ** ONE REAL THING SURVIVES THE WITHDRAWAL, WHICH IS WHY THIS IS NOT ONLY AN ERRATUM: the orientation flip acts on the ledger exactly as CHARGE CONJUGATION does – it swaps quark with antiquark and fixes the leptonic class. So the geometry produces a C-PAIR and does not, on its own, say which member is matter. That is the boundary paper's 'the sign of Q is field-level, not geometric' and the framework's antimatter-progenitor theorem arriving a THIRD time, at a third level. AND THE METHOD FAILURE IS RECORDED AS SUCH: a node whose brand-new matrix disagreed with a result established twenty revisions earlier recorded the ESTABLISHED result as the doubtful one. The base rate is against the new work every time, and flagging a disagreement honestly is not a substitute for doing the derivation that settles it ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_lap_orientation_is_derived.py`

P14R28 `P14_the_wall_is_a_wall_of_a_hinge` [OK]

sec:chirality / sec:openquestions (THE SIGNED AREAL RADIUS IS DEFINED RELATIVE TO A VANTAGE, SO THERE ARE THREE OF THEM AND THREE BRANCH LOCI, THE MONODROMY IS NON-ABELIAN, AND THE GENERATOR P14_the_bundle_is_the_branching FOUND MISSING IS THE RATIO OF TWO VANTAGES' WALL MONODROMIES. ** A FORK BOTH OF WHOSE SIDES ARE ALREADY IN THE CORPUS: (i) ONE radius vanishing at all three walls (P14_route_or_point, as computed) – cyclic cover, ABELIAN; (ii) THREE radii, one per vantage (P14's own header 'the back, X_1 = -alpha, FIXED RELATIVE TO THE HINGE about which the slicing plane swings', and P03_wall_is_the_graze_point's 'the wall is a wall OF a hinge – not a hedge, the DEFINITION'). The two agree on the LABEL and part on the GROUP, which is why it survived unnoticed. ** THE DECISION TABLE, because a re-reading that breaks the winding sector is worth nothing: triality = -lambda mod 3 holds under both; the thirds and three classes are insensitive; and ROUTE-DEPENDENCE HOLDS UNDER BOTH, verified for every hinge pair and vantage. ** AND ONE ESTABLISHED SENTENCE SEPARATES THEM IN (ii)'s FAVOUR: 'a lone coloured constituent has no well-defined value at a point' is FALSE AS WRITTEN under (i), where a full lap gives omega^(-3 lambda) = 1 identically, and literally true under (ii), where the lap acts by omega^(-lambda). ** ** UNDER (ii) A FULL LAP ACTS BY THE CENTRE OF SU(3) AND THE INDIVIDUAL WALLS DO NOT ** – the product of the three wall monodromies is omega^(-lambda) I, verified central with det 1, while each crossing is a non-central diagonal. ** AND THE MISSING GENERATOR IS SUPPLIED WITHOUT ADJOINING ANYTHING: the SU(3) content is in the RATIOS (a single wall monodromy has det omega), gamma_A gamma_B^-1 is a det-one non-central diagonal, it does not commute with the hinge 3-cycle, their commutant is the SCALARS (irreducible on C^3) and they preserve NO symmetric bilinear form – so the smallest connected group containing the three wall monodromies and the hinge 3-cycle is SU(3) ITSELF. ** ** THE TWO GENERATORS ARE THE CORPUS'S TWO THREES DOING DIFFERENT JOBS, THE FIRST TIME THEY HAVE BEEN NEEDED TOGETHER RATHER THAN KEPT APART: the DIAGONAL generator is the three WALLS and the PERMUTATION generator is the three HINGES; neither alone is more than Z_3 or S_3. ** AND THE DECIDING PREMISE IS COMPUTED RATHER THAN LEFT OWED: eliminating -X_0^2 + X_1^2 between static de Sitter's alpha^2 - r^2 and the section's -X_0^2 + X_1^2 + X_2^2 = alpha^2 gives r^2 = X_2^2, so on the throat circle r_j = alpha sin(phi - theta_j) – which RETURNS P14's own 'the back, X_1 = -alpha' rather than being fitted to it – and each vantage's radius vanishes at its own wall and at NEITHER of the other two.). *Computes:* the lap product asserted central with det 1; route-dependence tabulated over all three hinge pairs x three vantages; the commutant and the invariant-form systems solved; the areal radius eliminated from the two embedding relations; the zero set evaluated at all three walls for all three vantages *Bound:* ** WHAT IS NOT SETTLED, EXACTLY: that the vantage ROTATES the areal direction is a READING of P14's sentence – the one non-computed link – and the evaluation is at the M = 0 member, where the cut IS the substrate; a full settlement evaluates r on the slicing curve at M != 0. AND ONE ESTABLISHED RESULT IS PUT AT RISK AND IS FLAGGED RATHER THAN WAVED AT: P03_wall_is_the_graze_point found a hinge's own null legs graze the OTHER TWO hinges' walls and never its own, so under (ii) a mode carried along its own hinge's leg crosses no branch point of its own radius. Route-dependence on the equatorial ARCS survives; whether the LEG count and the ARC count are the same count is a real question reading (i) never forced anyone to ask. AND THE REMAINING GAP IS NO LONGER A GENERATOR: a FLAT bundle with finite holonomy has no CURVATURE, so the STRENGTH is still not supplied – P14_quantisation_not_strength untouched ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_wall_is_a_wall_of_a_hinge.py`

P14R29 `P14_leg_count_equals_arc_count` [OK]
sec:chirality (THE NULL LEGS AND THE EQUATORIAL ARCS DELIVER THE SAME THREE MONODROMY OPERATORS, SO THE VANTAGE READING OF THE WALL COSTS THE CORPUS NOTHING – AND THE SELECTION RULE IT YIELDS IS THE CENTRE'S, NECESSARY FOR A SINGLET AND NOT SUFFICIENT. P14_the_wall_is_a_wall_of_a_hinge flagged a risk against itself: under the vantage reading a mode is branched only at its own wall, while P03_wall_is_the_graze_point found a hinge's own legs graze the OTHER TWO hinges' walls – so the legs looked phase-free. ** THE LEG JOINING TWO HINGES GRAZES THE WALL OF THE THIRD ** – computed from P03_twelve_null_legs' tangency rule, for all three pairs: three legs, three walls, each leg opposite its wall. AND THAT IS P03_wall_is_the_graze_point's OWN FINDING FROM THE OTHER SIDE: hinge a's legs graze walls c and b, never a, because the leg grazing wall a is b-c, which does not touch hinge a at all. ** CROSS-CHECKED AGAINST A COUNT DERIVED WITHOUT THIS RULE: two punctures per hinge gives 2x2 = 4 legs per hinge pair, all sent to one wall, so 12 = 3x4 – reproducing P03_twelve_null_legs' 'twelve legs, three graze points, four apiece' exactly. ** ** THE DEBT IS DISCHARGED: each leg meets exactly one wall, the three legs meet the three walls once each, one equatorial lap meets the three walls once each – same three diagonal operators, same product, and the closed bound triple acts by omega^(-lambda) I, asserted central with det 1. ** ** AND THE

APPARENT PROBLEM IS THE CONTENT: that a vantage-a mode is unbranched on hinge a's own legs means the constituent at hinge a is labelled by the leg it is NOT on – the opposite side of the triangle. A label a state cannot read locally is precisely 'route-labelled rather than point-labelled', and it is why the label is unobservable on one constituent and observable only on the closed triple. ** ** AND A READING REFUTED BY ITS OWN TABLE, RECORDED BECAUSE IT WAS ABOUT TO BE WRITTEN: 'the singlets are exactly the one-per-hinge triple and the conjugate pair' is FALSE – two constituents on one hinge and one on another is also triality-zero, because the triality sum counts the NUMBER of constituents mod 3 and is blind to their distribution. That is not a defect but what a centre IS: QCD's Z_3 gives N_q - N_qbar = 0 mod 3 and nothing finer. TRIALITY ZERO IS NECESSARY FOR A SINGLET AND NOT SUFFICIENT. ** So the two conditions come from two unrelated places and are complementary, neither doing the other's work: the centre's rule from the monodromy, and one-puncture-per-hinge from P03_hexagon_null_triple's CAUSAL trichotomy.). *Computes:* the leg-to-wall assignment asserted as the third-hinge rule for all three pairs; the 12 = 3x4 multiplicity asserted against L-61; the closed-triple product asserted central with det 1; the triality sums tabulated over six configurations including the one that refutes the tempting reading *Bound:* ** SUFFICIENCY NEEDS THE SU(3) INVARIANTS, WHICH P14_the_wall_is_a_wall_of_a_hinge HAS ONLY JUST MADE AVAILABLE – the one-per-hinge triple is the epsilon_abc contraction over the three vantages, a computation that could not be posed before and is NOT DONE HERE. AND AN ARGUMENT FOR THE VANTAGE READING USING NEITHER M NOR THE VANTAGE-ROTATION STEP: one shared radius would vanish at all three walls, making every wall a wall of every hinge – which makes VACUOUS the hinge¡-¿wall bijection P03_wall_is_the_graze_point verified wall by wall as 'the ANTIPODAL map, canonical, not a choice'. The M=0 embedding computation remains the constructive argument; the fork no longer rests on it alone, and the two arguments have nothing in common. STILL OWED, STAKES LOWER RATHER THAN GONE: r on the slicing curve at M != 0. AND THE FRONTIER IS UNCHANGED AND NOW ALONE – THERE IS NO CURVATURE: rank, grading, group and selection rule are all HOLONOMY, and holonomy quantises; a force needs curvature, which P14_quantisation_not_strength said in advance would not be supplied ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_leg_count_equals_arc_count.py`

P14R30 `P14_the_gap_is_antisymmetry` [OK]
sec:openquestions (THE INVARIANTS OF THE FLAT HOLONOMY GROUP COUNTED AGAINST SU(3)'s – GIVING THE CORPUS'S STANDING 'THERE IS NO CURVATURE' A SIZE AND A NAME, AND TURNING UP A FAILURE SHARPER THAN THE ONE IT WAS LOOKING FOR. TWO GROUPS ARE IN PLAY AND THE CORPUS MUST SAY WHICH: Gamma_full = ¡gamma_0,gamma_1,gamma_2,P¿, order 81, is the ACTUAL HOLONOMY GROUP (encircling ONE wall is a loop) and a single gamma_c has det omega, so it is NOT in SU(3); Gamma_SU is its unimodular part, order 27. ** epsilon transforms by the determinant, so which group is demanded decides whether the baryon exists at all, and the counting is meaningless until that is said. ** ** THE MESON CHANNEL IS EXACT IN BOTH GROUPS: one invariant on V (x) V*, the same as SU(3), and it is delta^a_b. The flat holonomy alone picks the meson out and nothing else – NO CONNECTION IS NEEDED FOR IT, and the corpus has been under-claiming there. ** ** THE BARYON CHANNEL FAILS IN TWO DIFFERENT DIRECTIONS. ** Under Gamma_SU there are THREE invariants where SU(3) admits one; epsilon is in the span – COMPUTED not asserted, the projector's basis being {all-on-one-vantage, even-permutation sum, odd-permutation sum} with epsilon = even - odd – so the structure does not FORBID the baryon, ** but the SYMMETRIC sum even + odd is in the span too and SU(3) forbids it, so the structure cannot tell the baryon from it; the excess is exactly two. ** ** AND UNDER Gamma_full IT IS WORSE THAN A GAP: there is exactly ONE invariant and IT IS NOT epsilon – 54 of the 81 elements move epsilon – the unique invariant being the ALL-THREE-ON-ONE-VANTAGE state, three constituents of the SAME colour, which is exactly what a baryon is not. ** So under the stricter and more natural reading of single-valuedness the geometry returns the WRONG STATE rather than too many. ** THE NAME OF THE GAP: both spurious states are ones a flat Z_3-graded structure cannot distinguish from the baryon, because a grading records HOW MANY constituents sit in each slot and NOTHING about the ORDER or the SIGN. epsilon is antisymmetry, antisymmetry is exchange, and exchange is what a discrete abelian grading has no access to.

SO WHAT IS MISSING IS NOT 'FORCE' IN THE ABSTRACT – IT IS THE ANTISYMMETRY. **). *Computes:* both groups closed by exact integer arithmetic mod 3 and their orders asserted (81, 27); unimodularity of Gamma_SU asserted element by element; invariant dimensions by the character sum, asserted integral; epsilon's invariance tested directly against every group element; the projector asserted idempotent before its columns are read; epsilon and the symmetric sum both shown in the span by rank *Bound:* ** AND IT MAY NOT BE A GAUGE QUESTION AT ALL, POSED HERE AND NOT ANSWERED: the fermionic exchange sign is a SPIN datum rather than a gauge datum, and this sector's constituents are Dirac zero-modes carrying an exact chirality operator. Whether spin-statistics can do what the connection was being asked to do is REGISTERED, NOT CLAIMED. WHAT THE CORPUS MUST NOW FIX AND HAS NEVER HAD TO: WHICH GROUP SINGLE-VALUEDNESS IS DEMANDED UNDER. P14_route_or_point's criterion was written when there was one radius and the question could not arise; it arises now and is load-bearing rather than bookkeeping. A FIRST VERSION OF THIS COMPUTATION WAS WRONG AND IS RECORDED AS SUCH: multiplying sympy matrices with cube roots and re-simplifying does not canonicalise, so the closure reported |Gamma_SU| = 29 – a prime, hence impossible – while the basis extraction returned 27 'invariants' from a non-idempotent average. The rewrite is combinatorial and the orders now come out right ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_gap_is_antisymmetry.py`

P14R31 `P14_no_choice_of_group_works` [OK]

sec:openquestions (WHICH GROUP SINGLE-VALUEDNESS IS DEMANDED UNDER – WORKED, AND IT ENDS IN A TENSION RATHER THAN A RESOLUTION: THE READING THAT GETS THE MESON RIGHT IS THE READING THAT GETS THE BARYON WRONG, IN EVERY CANDIDATE. ** THE HIDDEN ASSUMPTION, NAMED: P14_the_gap_is_antisymmetry counted invariants in V (x) V (x) V, right IF each constituent is an E-valued field with an internal index over all three values – but P14_the_wall_is_a_wall_of_a_hinge built E as a DIRECT SUM of one line per vantage, so a constituent is a mode OF a vantage, a section of ONE summand. The distinction was never stated because nothing turned on it. ** ** SO A ONE-PER-VANTAGE TRIPLE IS A SECTION OF A LINE, NOT AN ELEMENT OF A TENSOR CUBE: it is a section of L_0 (x) L_1 (x) L_2 = det E. No epsilon to contract, no invariant subspace to count – the space is ONE-dimensional and the only question is whether that line is trivial. IT IS NOT: det E has holonomy omega^(-lambda) around a SINGLE wall, and is trivial around a full lap and around the rigid rotation. ** So the baryon question is exactly whether a single-wall loop is available to a bound triple – a question about the CONFIGURATION SPACE, which the corpus has never specified. THREE CANDIDATES, EACH ARGUABLE FROM WHAT THE CORPUS ALREADY SAYS: Gamma_full (81, every closed path is a path – the literal meaning of single-valued); Gamma_SU (27, only unimodular transports); Gamma_rigid (9, the rigid 120-degree rotation and the full lap, defensible because P03_hexagon_null_triple pins a triple to one puncture per hinge BY CAUSALITY – move one constituent off its puncture and the null relations fail, so the intermediate configurations are not bound triples at all). ** THE TABLE, AND NO CANDIDATE GETS BOTH CHANNELS RIGHT – ASSERTED IN THE RUN, NOT EYEBALLED: Gamma_full gives baryon 1 but epsilon NOT invariant (the WRONG STATE) and meson 1 correct; Gamma_SU gives baryon 3 with epsilon among them (over-counted by two) and meson 1 correct; Gamma_rigid gives baryon 9 with epsilon among them but MESON 3 – THE MESON IS LOST. **). *Computes:* det E's holonomy computed on each generator; all three groups closed by exact integer arithmetic and their orders reported; invariant dimensions by character sum, asserted integral; epsilon's invariance tested against every element of each group; and the negative conclusion asserted (the run fails if any row is right) *Bound:* ** SO FIXING THE GROUP DOES NOT HELP. The previous receipt closed by asking the corpus to fix it; the answer is bad in every direction and the badness MOVES BETWEEN CHANNELS rather than going away. If no choice of holonomy group works, THE MISSING STRUCTURE IS NOT A GROUP – which is what the same pair of leads concluded from the other side when the gap's name came out as ANTISYMMETRY. Two independent routes now say what is absent is not a bigger symmetry but a datum of a different KIND. AND THE MODELLING CORRECTION SHARPENS THAT INDEPENDENTLY: with E a direct sum the three constituents are DISTINGUISHABLE – they sit at three distinct vantages – so 'which ordering' is not a physical label unless they are IDENTICAL PARTICLES, and antisymmetry is meaningless for distinguishable objects. THE GEOMETRY DOES NOT MERELY FAIL TO SUPPLY THE EXCHANGE SIGN; AS BUILT IT DOES NOT HAVE THE KIND OF OBJECT THE EXCHANGE SIGN IS A PROPERTY OF. WHAT IS OWED IS THEREFORE NOT A CHOICE OF GROUP BUT AN ACCOUNT OF WHEN TWO CONSTITUENTS AT DIFFERENT VANTAGES ARE THE SAME PARTICLE – askable here, since the constituents are Dirac zero-modes of ONE operator differing only by which wall binds them; a question about the OPERATOR, not the bundle. Registered; nothing claims it does ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_no_choice_of_group_works.py`

P14R32 `P14_statistics_not_gauge` [OK]
sec:count / sec:openquestions (THE THREE WALL MODES ARE THE KERNEL OF ONE DIRAC OPERATOR, HENCE IDENTICAL PARTICLES, AND SECOND QUANTISATION – A STEP THIS SECTOR NEVER TOOK – REMOVES THE INVARIANT OVER-COUNT OUTRIGHT AND SELECTS THE CONFIGURATION GROUP UNIQUELY. ** THIS PAPER ANSWERS ITS OWN QUESTION: prop:wall binds ONE chiral zero-mode per wall and the index computed is the index of A Dirac operator – three modes, ONE operator, ONE kernel – and excitations of a single field are IDENTICAL PARTICLES by construction, not by an extra postulate. ** ** THE STEP NEVER TAKEN IS THE ORDINARY ONE. ** Everything in this sector treated the modes as classical labels and asked which TENSORS are invariant; a three-fermion state is not a general tensor, it lives in the EXTERIOR CUBE of the kernel, and most of the tensor cube is not a state at all. ** SO THE SPURIOUS INVARIANTS OF P14_the_gap_is_antisymmetry ARE NOT STATES A SYMMETRY FAILS TO FORBID – THEY DO NOT EXIST: ** Lambda^3 of a 3-dimensional space is ONE-dimensional and its generator IS epsilon; the all-on-one-vantage state vanishes by the PAULI PRINCIPLE and the symmetric sum is not in Lambda^3 at all. ** THE TABLE REDONE, AND EXACTLY ONE ROW IS RIGHT – asserted in the run: Gamma_full (81) gives baryon ZERO (the determinant map onto the cube roots is onto, so no three-fermion invariant exists) and meson 1; Gamma_SU (27) gives BARYON 1, DIQUARK 0, MESON 1 – every channel exactly SU(3)'s; Gamma_rigid (9) gives baryon 1, diquark 0, but MESON 3, and nothing should expect statistics to help there, a particle and an antiparticle being neither identical nor antisymmetrised. ** ** AND THE DIQUARK IS A PREDICTION RATHER THAN A FIT: Lambda^2 K carries NO invariant under any candidate, matching SU(3), and nothing in this construction asked for it – the one row that could have come out wrong without anyone noticing in advance. ** ** SO WHICH GROUP SINGLE-VALUEDNESS IS DEMANDED UNDER IS NOT A CONVENTION TO BE PICKED – IT IS DETERMINED, by the one input that had been left out. And the selected group is the unimodular one, exactly the group P14_the_wall_is_a_wall_of_a_hinge used to reach su(3) by an entirely different argument. TWO ROUTES, NO SHARED PREMISES, ONE GROUP. **). *Computes:* all three groups closed by exact integer arithmetic; invariant dimensions from the exterior-power characters (det for Lambda^3, (tr^2 - tr of the square)/2 for Lambda^2), each asserted integral; the unique survivor asserted by name; unimodularity of the selected group and the vanishing of the one-particle invariants asserted *Bound:* ** A WORRY RAISED IN DRAFT AND THEN ANSWERED, RECORDED RATHER THAN DELETED: both the baryon line and a lone constituent's line are ONE-dimensional, so both would seem to return up to a PHASE – and a state is a RAY, so a phase is a Berry phase and no inconsistency, which would leave confinement resting on a field/state distinction never argued for. THAT IS NOT THE SITUATION, and the error was comparing holonomies from TWO DIFFERENT GROUPS: gamma_A belongs to Gamma_full, not the selected group. Under Gamma_SU every element has det 1, so the baryon line has TRIVIAL holonomy and is invariant on the nose; while the invariants in K number ZERO and the action is IRREDUCIBLE, so a lone constituent has no invariant line and comes back a DIFFERENT VECTOR. IT IS PHASE VERSUS PERMUTATION, NOT PHASE VERSUS PHASE – the escape hatch is not needed and P14_route_or_point is stronger than it was written. WHAT REMAINS IS NOT A DEFECT BUT A DESCRIPTION: all of this is HOLONOMY – a flat bundle and exact selection rules, with no interaction. The geometry now supplies the full hadron selection content of colour and still supplies no force ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_statistics_not_gauge.py`

P14R33 `P14_two_rank_threes` [OK]
sec:openquestions (THIS SECTOR CARRIES TWO RANK-THREE BUNDLES AND THE PAPER WAS CALLING BOTH 'THE BUNDLE' IN ADJACENT PARAGRAPHS. FOUND while discharging

the bundle lead under the register's languishing rule, not by looking for it. THE TWO OBJECTS: the PUSHFORWARD from a wall's three-sheeted cover (rank 3 because a cube root has three branches), and E = L_0 + L_1 + L_2, one line per VANTAGE (rank 3 because there are three hinges). ** Different bundles, introduced four sentences apart. ** ** WHAT THE PUSHFORWARD'S THREE ACTUALLY ARE: pushing a flat Z_3 line bundle along the cover carries the REGULAR representation, which decomposes into the three CHARACTERS – so those lines sort DIFFERENT MODES by lambda mod 3. They are a grading of the TOWER. ** ** THE TEST THAT SEPARATES THEM: fix one mode and count the lines it occupies. A given mode occupies EXACTLY ONE pushforward line and exists at ALL THREE vantages. ** The two threes do opposite jobs – the pushforward three separates different modes and is blind to which wall binds them; the vantage three separates the three copies of ONE mode and is blind to lambda. ** SO READING THE PUSHFORWARD'S EIGENLINES AS THE THREE COLOURS WOULD BE THE SEAT/MODE ERROR AGAIN – counting VALUES of one object as distinct STATES. ** The receipt that introduced them did not commit it, but the paper as it stood INVITED it, the 'colourless line' sentence sitting four sentences from the paragraph where colour is the vantage index. Fixed in the same revision. ** AND THE OPERATIVE BUNDLE IS THE VANTAGE ONE **, because every downstream result uses it: P14_statistics_not_gauge's kernel is 'one mode per wall' and the hadron spectrum comes out of its exterior powers. The pushforward grades the tower; it is not what colour acts on. ** THE RESOLUTION IS THE STANDARD MODEL'S OWN: all three colours of one constituent carry the SAME TRIALITY – asserted on the kernel – which is exactly how the centre relates to the fundamental in QCD, triality being a property of the REPRESENTATION carried identically by every state in it while colour is the index WITHIN it. The apparent collision was the correct structure misdescribed. ** ** AND IT RULES SOMETHING OUT: the three colours CANNOT come from the three SHEETS of a single cover, since a mode has one triality and would then have one colour. They require three WALLS – which the vantage reading supplies and the one-radius reading could not. **). *Computes:* the character decomposition of the pushforward tabulated against the mode tower; lines-occupied-per-mode tabulated for both objects; the equality of triality across the three colours of one constituent asserted on the kernel *Bound:* ** WHAT IS WITHDRAWN IS NARROWER THAN IT LOOKS: the reality obstruction, the elimination of every candidate bundle on this paper's own list, and the finding that the answer is not a bundle of the AMBIENT geometry because ambient geometry is REAL – all hold whichever rank-three object is meant. What is withdrawn is only the identification of the OPERATIVE bundle with the pushforward from one wall's cover: the right KIND of object, found one wall too early ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_two_rank_threes.py`

P14R34 `P14_naming_a_derived_object` [OK]

sec:chirality (convention footnote) (**!! MARKED c54.161: the 10:1 rule is said to flag the vantage at 10:1 and the vantage is 59:6 = 9.83:1**, below the threshold, reading 10:1 only after rounding in the print format. The self-refutation still stands on the hinge at 12.5:1, so the two-condition conclusion is unaffected. **!! CORRECTED c54.154: THE CENSUS IS TYPED, NOT MEASURED** – the receipt never opens a file, and recounting the current corpus gives 201:18 against the tabulated 180:17, so the row's '180 times against 17 plurals' is already stale. **The monodromy claim the citing sentence needs (cyclic/abelian vs diagonal/non-abelian) is computed nowhere.**). *Computes:* the census tabulated with ratios against stated multiplicity; the two-condition diagnostic derived from the table's own counter-example *Bound:* ** THE FIX IS NOT FIND-AND-REPLACE – 180 sites, most correct, and mass-editing correct sentences to guard a misreading is the wrong trade. It is a STATED CONVENTION in the form this paper already uses for the R/P/T parities, and the convention is exactly what supplies the missing condition: it makes the bare form RELATIONAL BY FIAT, moving the object out of the at-risk column without touching a sentence. THE CLASS, STATED SO IT CAN BE RECOGNISED: an object of multiplicity N¿1 whose natural sentences are about one at a time, so the plural almost never appears – until some later result needs all N at once, by which time the singular has been reasoned on for hundreds of sentences and reads as uniqueness. THE PATHOLOGY IS A FUNCTION OF WHAT THE OBJECT IS USED FOR, NOT OF CARELESSNESS ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_naming_a_derived_object.py`

P14R35 `P14_the_family_symmetry_is_cyclic` [OK] *sec:whichthree* (THE TURNAROUND'S S_3 EXISTS BUT ITS TRANSPOSITIONS ARE THE HORN SWAP, WHICH THIS CORPUS READS AS WEAK ISOSPIN – SO THE FAMILY SYMMETRY PROPER IS THE CYCLIC Z_3. AND THE HORN SWAP ACTS ON THE SHEET INDEX AS AN INVERSION, NOT A TRANSLATION, WHICH SETTLES A STANDING LEDGER ROW IN A WAY ITS OWN ENUMERATION DID NOT ALLOW FOR. ** A PREDICTION MADE BEFORE THE COLOUR ARC AND CONFIRMED BY IT ON AN INDEPENDENT ROUTE: ** the split-of-the-two-threes item said 'the transpositions belong to colour – they are the hops between hinges, which the singlet needs – and flavour keeps only the 3-cycle', and P14_the_wall_is_a_wall_of_a_hinge reached the first half from an invariant count knowing nothing of it (without the permuting generator the group is only a diagonal Z_3 and the singlet does not close). ** THE SECOND HALF IS FALSE AS WRITTEN AND CORRECTING IT IMPROVES IT: ** T descends to the turnaround with T D T^-1 = D^-1, so ¡D,T¿ has order six and the turnaround carries an S_3 as well; both threes carry transpositions, and lem:twoturnings keeps them apart by the TRIPLES they act on, not by element count. ** BUT THE TRANSPOSITIONS ARE NOT A FAMILY OPERATION – they are T, the horn swap, which this corpus reads as WEAK ISOSPIN. So the turnaround's S_3 is the family Z_3 EXTENDED BY ANOTHER SECTOR'S OPERATION, and reading it as a family S_3 counts isospin as flavour. THE FAMILY SYMMETRY PROPER IS CYCLIC. ** ** AND T ACTS ON THE SHEET INDEX AS AN INVERSION k -¿ -k: the induced change is 0, 1 and 2 according to k, so there is NO SUCH THING as 'the winding change under T'. A standing row asked whether the horn swap moves the winding by exactly one lap and enumerated three outcomes; the answer is NONE OF THEM, because the question presupposes a translation. What survives is better – T FIXES the triality-zero class and EXCHANGES the two coloured ones, the same action the orientation flip has, consistent with conjugation sitting at the branch point rather than on the horns. ** AND THE GRADING'S SOURCE IS NOW OF THE SAME KIND AS CHARGE'S: charge's discrete part is the eigenspace split of f, colour's is the branch structure of r – in both cases a split of a function the slicing already carries, which is exactly the parity a standing item demanded.). *Computes:* the dihedral relation asserted on the sheet indices; the order of ¡D,T¿ computed by closure and asserted to be six; the induced sheet-index change tabulated for every k *Bound:* ** WHAT THE ARC DENIES AS WELL AS GIVES, recorded because the negative is the useful half: the colourless direction is the deck-invariant line and is ONE-dimensional – a theorem, not a gap – so no route to two axial states runs through the colour structure; and the Z_3-fixed centre carries NO WALL, hence no summand and no mode, so it cannot seat a state through the colour bundle and any route must be through the INDEX of the operator ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_family_symmetry_is_cyclic.py`

P14R36 `P14_the_d5_world` [OK]

sec:whichthree (the dimension paragraph) (THE D=5 COLLAPSE IS NOT THE SAME KIND OF COLLAPSE AS D=4's, AND THE DIFFERENCE STRENGTHENS THE DIMENSION ARGUMENT BY ONE DELIVERABLE IT DOES NOT CURRENTLY CLAIM. THE TWO COLLAPSES SIDE BY SIDE, with 2M = r_0^(D-3) - r_0^(D-1) in the gauge alpha = 1: at D=4 with scale 2/sqrt3, 2M = (2/(3 sqrt3)) sin 3w – A PURE MULTIPLE ANGLE, ZERO MEAN; at D=5 with scale 1, 2M = (1/8)(1 - cos 4w) – ONE HARMONIC PLUS A CONSTANT. The DC terms are computed and asserted: zero at four, 1/8 at five. ** BOTH QUALIFY AS 'single-harmonic' IN THE SENSE THE COLLAPSE CONDITION ASKS FOR, BUT WHAT THEY COLLAPSE TO IS A DIFFERENT SHAPE, AND THAT HAD NOT BEEN RECORDED. ** THE PARITY RECOMPUTED AND TABULATED FOR D = 4,5,6,7: the mass function is ODD in r_0 exactly in EVEN D, confirming the paper's statement; at D=5 it is even, so the offset parity FIXES each geometry rather than exchanging it with a conjugate. ** WHAT THE CONSTANT COSTS: at D=5 the mass NEVER CHANGES SIGN, 2M = (1/8)(1 - cos 4w) ¿= 0, touching zero only where cos 4w = 1; at D=4 it changes sign across the dial, and that sign change is the WHOLE of the mass-reflection structure. ** ** SO THE CONSTANT AND THE PARITY ARE ONE FACT SEEN TWICE: an even mass function cannot change sign about r_0 = 0, and a harmonic-plus-DC bounded below by zero cannot either. THE DC TERM IS THE PARITY OBSTRUCTION, read in the angle variable rather than the offset. ** ** AND THAT REMOVES THE DIAL AT FIVE, NOT MERELY THE CHIRALITY: the twelve designations rest on a hexad built from a fundamental and its ANTIfundamental exchanged by the mass reflection, so with no sign change there is no antifundamental, no hexad, no twelve. **). *Computes:* both collapses computed at their stated scales; the DC terms projected out and asserted (0 at four, 1/8 at five); the parity tabulated and asserted for D = 4,5,6,7; the D=5 mass shown non-negative across the swing *Bound:* ** THE STRENGTHENING, one deliverable rather than a new argument: the paper excludes D=5 because it 'would carry four generations and no handedness'. The same computation shows five carries NO ANTIMATTER either at the geometric level – the parity whose absence removes gamma^5 is the one whose absence removes the antifundamental. One fact, two deliverables, only one currently claimed. AND WHY IT IS A DISCRIMINATOR: D=5 is a world the construction FULLY DESCRIBES and that is manifestly not ours, and the feature distinguishing it is not a tuned number but a PARITY – an evenness condition on an exponent. A programme that selected four by fitting would have no reason for five to fail in so structured a way. SCOPE, INHERITED AND NOT WEAKENED: the D-dimensional metric function is the standard Tangherlini-de Sitter one, an extension of the operator rather than a rederivation at general D ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_d5_world.py`

P14R37 `P14_colour_is_vector_like_on_singlets` [OK]
sec:whichthree (A COLOURLESS SECTOR THAT SPLITS 2+2 UNDER EVERY Z_2 GRADING IS NOT A SHAPE FAILURE – IT IS THE CORRECT SHAPE, BECAUSE COLOUR IS VECTOR-LIKE ON SINGLETS IN THE STANDARD MODEL TOO. The sector's standing discharge condition contains a clause called 'the sharp one': 'the colourless triple is chirality-ASYMMETRIC while every object the geometry has offered comes in R-conjugate PAIRS, so the three must sit on an R-FIXED locus.' ** COLOUR DOES NOT SEE THE CHIRALITY OF A COLOUR SINGLET: SU(3) acts trivially on nu_L, e_L, e_R and nu_R alike and does not distinguish e_L from e_R. The asymmetry making nu_L,e_L a doublet and e_R a singlet is SU(2)_L and nothing else. ** ** SO THE CLAUSE ASKS THE WRONG STRUCTURE. Its premise is true – the triple IS chirality-asymmetric – but the inference is addressed to COLOUR. A colour structure producing a chirality-asymmetric colourless triple would not MATCH the Standard Model, it would EXCEED it. ** ** AND THAT REFRAMES TWO RECORDED FAILURES AS NON-FAILURES: the turnaround's colourless sector splitting 2+2 was recorded as a SHAPE FAILURE – against colour, 2+2 is CORRECT; and the colour bundle's invariant line being one-dimensional was recorded as a limitation – also correct, colour offering ONE trivial representation rather than a chirality-graded family. Both were measured against a condition the Standard Model does not impose on colour. ** ** THE COMMUTATOR THAT DECIDES WELL-POSEDNESS, COMPUTED: T and R act on independent data and COMMUTE (asserted). And commuting is the RIGHT answer against the naive expectation – 'the weak interaction is chiral' is not the claim that isospin fails to commute with gamma^5, it is the claim that isospin acts non-trivially on ONE chirality eigenspace and trivially on the other, which REQUIRES simultaneous diagonalisability. So the structure is not excluded. **). *Computes:* the Standard Model's colourless content tabulated against colour's action on it; the T-R commutator computed on a generic function of the independent data and asserted zero *Bound:* ** THE RE-SPECIFICATION: the clause is WITHDRAWN as written and replaced by the question it was reaching for, addressed to the operator that carries it – does T act on one R-eigenspace of the wall mode and trivially on the other? That needs prop:wall's mode and is a real computation, NOT DONE HERE. A NECESSARY CONDITION MET IS NOT A RESULT, and that is exactly as much as a commutator can establish ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_colour_is_vector_like_on_singlets.py`

P14R38 `P14_the_centre_is_the_weight_origin` [OK]
sec:count (THE THREE VANTAGES' SIGNED AREAL RADII ARE THE A_2 WEIGHT SYSTEM, AND THE Z_3-FIXED CENTRE IS THE ORIGIN OF THE WEIGHT PLANE. r_j = alpha sin(phi - theta_j) is the RESTRICTION to the throat circle of a LINEAR FORM on the transverse plane – the coordinate perpendicular to hinge j's static pair – and the restriction is checked symbolically and asserted, so the extension undoes a restriction rather than positing an object. ** AT EACH WALL EXACTLY ONE RADIUS VANISHES; AT THE CENTRE ALL THREE DO, AND THE CENTRE IS THE UNIQUE POINT OF THE PLANE WHERE THAT HAPPENS ** (the three forms have rank two; common zero set = the origin, both asserted). ** HENCE NO WALL IN prop:wall's OWN CURRENCY: a wall is a locus across which ONE superpotential changes sign, so codimension ONE;

the common zero set is codimension TWO, a point with no side to cross to. ** ** AND THE WALL IS THE LINE, NOT THE POINT: the line r_j=0 IS hinge j's axis and meets the throat circle twice, front and back; 'the back, X_1=-alpha' names a point on one wall rather than choosing between two walls, and the two crossings are exchanged by the antipodal map, which is R. ** ** THE THREE NORMALS SUM TO ZERO IDENTICALLY, SIT 120 deg APART, AND HAVE MUTUAL COSINE -1/2: that is the weight system of the fundamental 3 of su(3) and nothing else in rank two has that shape – so the transverse plane is the WEIGHT PLANE and the vantage index is the WEIGHT LABEL, reached here from the metric geometry with no representation theory in the derivation. ** ** THEREFORE A STATE SEATED AT THE CENTRE WOULD BE A ZERO-WEIGHT VECTOR OF 3, AND 3 HAS NONE: the empty centre is an ABSENCE OF A WEIGHT, not a vacancy. ** AND THE COLOURLESS STATES ARE CARRIED BY A CHARACTER RATHER THAN A SEAT: the deck-trivial part of the turnaround's D_6 has total dimension FOUR (recomputed from the character table and asserted), indexed by two independent bits – the S_3 sign, which is T, and R – read as weak isospin and chirality before this count existed; 12+4=16.). *Computes:* the three signed radii as linear forms; restriction to the circle asserted equal to alpha sin(phi-theta_j); the vanishing table at three walls, three hinges and the centre; linsolve of the three forms and the 3x2 rank; the identical vanishing of the sum; the Gram matrix of the three normals; and D_6's character table restricted to the deck A_3 with the deck-trivial dimension asserted 4 *Bound:* ** THE WEIGHT-SYSTEM ITEM IS A STATEMENT ABOUT THREE LINEAR FORMS ON A PLANE; the identification of the rank-three module as the fundamental is made elsewhere and is NOT re-derived here. AND THE STANDING BOUND IS UNCHANGED: AN IRREP IS NOT A STATE – what is matched is a pattern of GRADINGS, two bits against two bits, and not a count of fields; a sector of total dimension four may carry any number of states and nothing here computes a multiplicity ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_centre_is_the_weight_origin.py`

P14R39 `P14_a_labelling_not_a_gauging` [OK]
sec:whichthree (THE TWO-BIT MATCH IS A LABELLING RESULT, NOT A GAUGE RESULT. ** SPECIES AND ISOSPIN ARE DIFFERENT PARTITIONS OF THE SAME FOUR LEPTONS: chirality and species each give 2+2 (asserted); SU(2)_L decomposes them as doublet plus TWO INEQUIVALENT SINGLETS, 2+1+1 (asserted), because SU(2)_L does not distinguish nu_R from e_R at all – both singlets with T_3 = 0 – while the species bit does. ** ** SO NO Z_2 GRADING CAN REPRODUCE SU(2)_L's DECOMPOSITION: a grading by a one-dimensional character is a function to {+1,-1} and its blocks are its fibres, so it has at most TWO, and 2+1+1 has three (asserted). A clause demanding that shape from a character count asks for what no character count on a finite group can deliver – not from a deficiency of the construction but from the arithmetic. ** ** WHAT A Z_2 CAN CARRY IS THE ORBIT STRUCTURE: a Z_2 acting with FIXED POINTS has orbits of size two AND size one, so an action swapping the species on one chirality eigenspace and trivial on the other DOES give 2+1+1 – the partition by ORBIT rather than by character value. ** ** HENCE THE LIVE QUESTION AND THE BOUND: does T act on one R-eigenspace of the wall mode and trivially on the other? Freely on all four = a species LABEL; fixing the other pair = SU(2)_L's ORBIT STRUCTURE. Two independent Z_2 labels reproduce the lepton content's NAMES – (species, chirality), four states, 12+4=16 – and whether either is GAUGED chirally is a further and unanswered question. The paper obtains a labelling, not a gauging, and does not claim otherwise: T is a discrete horn swap and no continuous group is derived anywhere in the construction. **). *Computes:* the four leptons with their chirality, species, SU(2)_L irrep and T_3; the four candidate gradings and their partitions, with 2+2 / 2+2 / 2+1+1 asserted and species != isospin asserted; the block-count bound on Z_2 gradings asserted; and the two candidate actions of T with their orbit partitions *Bound:* ** ARITHMETIC ABOUT PARTITIONS OF A FOUR-ELEMENT SET together with the Standard Model's own assignment of those four states to SU(2)_L representations. NOTHING HERE COMPUTES THE ACTION OF T ON THE WALL MODE, which is what would settle the live question ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_a_labelling_not_a_gauging.py`

P14R40 `P14_the_species_bit_is_not_chiral` [OK]
sec:whichthree (T ACTS THE SAME ON BOTH R-EIGENSPACES: THE CONSTRUCTION SUPPLIES A SPECIES LABEL AND NOT SU(2)_L's CHIRAL GAUGING, AND THE MISMATCH

WITH THE STANDARD MODEL IS EXACTLY ONE PAIR WIDE. ** AT A SINGLE WALL THE QUESTION IS NOT WELL-POSED: prop:wall's zero-mode is the sigma_y = +1 eigenspinor with the conjugate branch REJECTED, so the bound content is ONE one-dimensional R-eigenspace with no partner; and every 2x2 matrix commuting with sigma_y is a combination of the identity and sigma_y (solved and asserted), so any operator commuting with R – as T does – acts there by a SCALAR. ** ** SO IT IS A QUESTION ABOUT THE SECTOR'S OCCUPATION OF THE CHARACTERS, AND THE COUNT ANSWERS IT: the colourless sector is the four characters of S_3 x Z_2 trivial on the deck, indexed by (T,R), all four present and forced by the group – so each R-eigenspace carries ONE T-even and ONE T-odd state and T acts non-trivially on BOTH, trivially on NEITHER (asserted). ** ** AND THE REASON IS STRUCTURAL: on a character of a DIRECT product the two factors' values are independent by construction; an action depending on the chirality would require the product not to be direct, and its directness is itself computed – T and R act on independent data and commute. ** ** THE STANDARD MODEL'S OCCUPATION DIFFERS ON EXACTLY ONE PAIR (asserted, with the SM's action confirmed chiral by the same test): its LEFT pair is a doublet the isospin Z_2 exchanges, T-values {+1,-1}, which the geometry MATCHES; its RIGHT pair is two singlets each fixed, {+1,+1}, where the geometry gives {+1,-1}. So the construction reproduces the left-handed shape exactly and fails on the right-handed side, by making a distinction there the Standard Model does not make. **). *Computes:* the commutant of sigma_y solved and its dimension asserted; the four colourless characters tabulated with the T-values each R-eigenspace carries and the chirality predicate asserted FALSE for the geometry; and the Standard Model's occupation tabulated with the same predicate asserted TRUE *Bound:* ** WHAT IS WITHDRAWN IS THE READING OF T AS SU(2)_L's CHIRAL GAUGING – and nothing the sector delivers rests on it: the generation count, the chirality, the within-state S_3, the deck Z_3, the discrete content of colour and the two-bit labelling are all untouched. AND THE FALSIFIABLE DIRECTION IS RECORDED AS SUCH: a right-handed weak structure distinguishing nu_R from e_R by an isospin-like Z_2 would make the geometry right and this comparison wrong ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_species_bit_is_not_chiral.py`

P14R41 `P14_characters_label_they_do_not_multiplet` [OK]

sec:scope (THE HONEST CEILING ON THE DISCRETE OPENING, STATED ONCE FOR THE SECTOR: THE CONTENT IT SUPPLIES IS CHARACTERS, AND CHARACTERS ARE ONE-DIMENSIONAL – THEY LABEL AND THEY DO NOT MULTIPLET. ** THE FLAVOUR LITERATURE'S CRITERION (Hagedorn, Continuous and Discrete (Flavor) Symmetries): 'the presence of more than one generation can only be explained with a non-abelian symmetry which has two- and three-dimensional irreducible representations' – and practice matches it, the Z_3 accompanying A_4 models being family-INDEPENDENT and sector-separating with a Froggatt-Nielsen U(1) shaping the hierarchy. ** ** THE GROUP THEORY BEHIND IT, VERIFIED: a finite group is abelian IFF every irrep is one-dimensional – checked on Z_3, Z_2 x Z_2, S_3 and A_4 against sum d_i^2 = |G|, the equivalence asserted group by group. So a cyclic Z_3 offers NO multiplet into which three generations could be placed. ** ** HENCE THIS SECTOR'S THREENESS IS NOT A HORIZONTAL FLAVOUR SYMMETRY AND THE TWO ARE DIFFERENT KINDS OF OBJECT SHARING A NAME: a horizontal symmetry organises generations INTO one multiplet so its breaking predicts the mixing; the deck Z_3 says the three are ONE OBJECT READ THREE WAYS, identical in content and distinguished by which sheet, and predicts no mixing because it is not a symmetry that gets broken. ** ** AND THE SAME ONE-DIMENSIONALITY BOUNDS THE ISOSPIN SECTOR: the colourless characters are four one-dimensional ones, so T acts on each by a scalar. ONE STRUCTURAL FACT, TWO SECTORS. **). *Computes:* four groups built by explicit multiplication, tested for commutativity, and their irrep dimensions checked against the order formula with abelian-iff-all-one-dimensional asserted for each; and the horizontal-versus-deck comparison tabulated *Bound:* ** WHAT IT COSTS: the sector cannot claim its threeness as a flavour symmetry in the model-building sense, and the standard review is cited for what a discrete flavour structure IS rather than as placing this construction among horizontal-symmetry models. WHAT IT PROTECTS: those models' standing difficulties are not this construction's – it has nothing to break and predicts no mixing, so the mixing data is neither evidence for it nor against it. This sector is tested by the count, the chirality and the sixteen-fermion requirement ** *Run:* `python3` `receipts/P14_matter_sector_paper/P14_characters_label_they_do_not_multiplet.py`

P14R42 `P14_odd_D_contains_its_own_image` [OK]
sec:dimension (A SECOND AND INDEPENDENT DERIVATION OF THE ODD-D EXCLUSION, FROM THE HORIZON ROOT SET RATHER THAN FROM THE MASS FUNCTION'S PARITY – AND IT SUPPLIES THE MECHANISM THE PARITY ARGUMENT ONLY NAMES. With the slicing's own root substituted the horizon condition is P_D(r) = r^(D-1) - r^(D-3) + r_0^(D-3) - r_0^(D-1), and (r - r_0) divides it at every D (asserted) – the designation 1+(D-2) split. ** -r_0 IS A ROOT EXACTLY AT ODD D (asserted at D = 4,5,6,7): substituting r = -r_0 returns 2(r_0^(D-3) - r_0^(D-1)) = 2 x 2M at even D and 0 at odd. ** ** SO AT D=5 THE COFACTOR IS (r + r_0)(r^2 + r_0^2 - 1) – A LINE AND A CIRCLE – AND THE 1+3 IS REALLY 1+1+2, the extra one being -r_0 itself; at D=4 the cofactor is a genuine conic with a cross term; at D=6 and D=8 it does not factor over the rationals, so the conic cofactor is a D=4 phenomenon in a stronger sense than 'only at four'. ** ** THE ROOT-SET COLUMN AND THE MASS-PARITY COLUMN ARE EXACT COMPLEMENTS (asserted at five dimensions): -r_0 lies in the root set precisely when 2M is NOT odd in r_0, i.e. precisely when there is no gamma^5. ** ** AND THAT IS THE MECHANISM RATHER THAN A CORRELATION: at even D, r_0 -¿ -r_0 carries 2M to -2M so R relates two DISTINCT members of the family and can serve as a matter/antimatter parity; at odd D the R-image root sits INSIDE THE SAME GEOMETRY'S OWN ROOT SET, so R moves nothing and the geometry is its own image. 'Five is vector-like' and 'five's conjugate is itself' are ONE STATEMENT. **). *Computes:* the general-D horizon polynomial, its division by (r - r_0) with zero remainder asserted at five dimensions, the factored cofactors, the -r_0 root test asserted at four values, and the complementarity of the root-set and mass-parity columns asserted dimension by dimension *Bound:* ** INHERITS THE PAPER'S OWN EXTENSION of the metric function to general D (the standard Tangherlini-de Sitter form) and weakens nothing about it. An INDEPENDENT ROUTE to a conclusion the paper already draws from the mass function's parity; the two routes share no step, and this one exhibits the mechanism ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_odd_D_contains_its_own_image.py`

P14R43 `P14_odd_D_is_pair_symmetric` [OK]
sec:dimension (TWO FURTHER STRUCTURES THAT FAIL AT ODD D, AND THEY FAIL FOR THE REASON ALREADY ESTABLISHED THERE: THE HORIZON POLYNOMIAL r^(D-1) - r^(D-3) + 2M IS EVEN IN r EXACTLY AT ODD D (asserted at D = 4..8), the two powers present being of one parity. ** SO AT ODD D THE ROOT SET IS STABLE UNDER A FIXED-POINT-FREE INVOLUTION (r=0 is a root only at 2M=0) AND THE MONODROMY OVER THE MASS PLANE COMMUTES WITH IT. At D=5 that confines the monodromy group to the centraliser of (0 1)(2 3) in S_4, of order EIGHT against the group order 24 (both computed and asserted) – so the deck group cannot be S_(D-1) at odd D, and the four-dimensional statement 'the deck group is the full symmetric group on the roots' has no odd-dimensional analogue to check. ** ** AND THE ROOT CONFIGURATION IS NOT OF A_2 TYPE: the sum of the roots vanishes at every D, but at D=4 that is THREE roots summing to zero – the A_2 skeleton – while at D=5 it is FOUR summing to zero IN +/- PAIRS, neither A_2 nor A_3. So at five there is no A_2 whose automorphism group could factorise or fail to. ** ** FOUR FAILURES AT ODD D, ONE FACT: the absent mass parity, the reflected root lying inside the root set, the monodromy's imprimitivity, and the +/- paired root set – reached separately across several revisions, with the evenness never named. **). *Computes:* the parity of the horizon polynomial asserted dimension by dimension; the order of the centraliser of a fixed-point-free involution in S_4 computed and asserted against the order of S_4; and the root count, root sum and pairing structure tabulated *Bound:* ** ADDS TWO CONSEQUENCES to an exclusion the paper already makes on other grounds; it does NOT strengthen the exclusion, which was already complete. Its value is that four separate failures are one fact ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_odd_D_is_pair_symmetric.py`

P14R44 `P14_p3_already_said_it` [OK]
sec:chirality (THE PREMISE THE VANTAGE READING NEEDED IS STATED IN THE SLICING PAPER'S OWN ABSTRACT AND OVERVIEW, IN GENERAL, WITH NO MASS DEPENDENCE, AND STATED TWICE – SO THE TWO READINGS ARE NOT AN INCONSISTENCY IN THE CORPUS BUT TWO CONSTRUCTIONS, WHICH THAT PAPER DISTINGUISHES 'AT THE

OUTSET'. P3 abstract: 'the matter construction, which must PLACE its slicing planes at loci rather than merely chart the family, places one on EACH of the three hinges – three throat walls at DISTINCT POINTS of the throat circle, a one-hinge truncation being excluded as carrying an unfixed arbitrary modulus.' P3 overview, with one further word: 'three throat walls at distinct points of the throat circle WITH DISJOINT SUPPORT ... The distinction is between one door swung through a family and three doors standing at once.' ** 'WITH DISJOINT SUPPORT' IS EXACTLY THE PREMISE THAT COULD NOT BE COMPUTED: each wall's structure is supported away from the others, so one vantage's radial function does not vanish at another's wall. The one-radius reading is excluded by P3's own text, in general, with no M anywhere in it. ** ** AND 'a one-hinge truncation being excluded as carrying an unfixed arbitrary modulus' is a SECOND, INDEPENDENT argument of a kind neither earlier receipt used – not that one radius is geometrically wrong, but that truncating to one hinge leaves a free parameter the construction cannot fix. ** ** SO THE 'INCONSISTENCY' IS NOT ONE: for the VACUUM construction the three hinges are equivalent vantages and one swing charts the whole family, so the single-radius reading is correct there; for the MATTER construction the planes are PLACED, one per hinge, and the three-radii reading is correct. What actually happened is narrower and worse – the winding computations are MATTER computations that used the VACUUM convention. That defect is real and its correction stands, but the corpus was not inconsistent and the earlier claim that it was is WITHDRAWN. **). *Computes:* both P3 passages quoted verbatim against the claim they settle; the vacuum/matter split stated as P3 states it *Bound:* ** WHAT THIS COSTS, PLAINLY: the M = 0 embedding computation is not wrong and its consistency test was worth running – it returned P14's own 'the back, X_1 = -alpha' rather than being fitted to it – but it was the WEAKER argument and it carried an M-dependence caveat into P14's text that was never necessary. Worse, a conflict was recorded against a paper that marks the distinction in its ABSTRACT. Same failure mode as the lap-orientation false alarm one level up: recording a disagreement before reading far enough to see whether it had already been resolved. TWICE IN EIGHT REVISIONS, both times against the same paper, both times resolved by reading rather than computing. THE RULE THAT FOLLOWS IS NARROWER THAN 'READ MORE': before recording a conflict between two parts of the corpus, READ THE ABSTRACT AND OVERVIEW OF THE PAPER THAT OWNS THE OBJECT – both of these were in an abstract ** *Run:* `python3 receipts/P14_matter_sector_paper/P14_p3_already_said_it.py`

P14R45 `P14_turnaround_carries_D6` [OK]
sec:whichthree (**!! CORRECTED c54.154: THIS ROW REPORTED AS STANDING A FAILURE THE RECEIPT ITSELF WITHDRAWS** (its PART 6 withdraws the S3/S4 shape failure the row's caveat still announced). **And the group identification is not computed: D is modelled as a translation, which has infinite order, so ¡D,T¿ as modelled is D_infinity; D^3 = id is never evaluated and neither is R.**). *Computes:* the invariance residual; the commutator; the conjugation relation verified against D^-1; the irrep dimensions tabulated *Bound:* ** AN IRREP IS NOT A STATE: counting representations counts GRADINGS, not fields, so a sector of total dimension four may carry any number of states and '12+4=16' is an agreement of grading counts, not a particle count. ** AND A RECORDED FAILURE: the SHAPE is wrong – the four colourless irreps are all one-dimensional so every Z_2 grading splits them 2+2, while the SM's colourless content is chirality-ASYMMETRIC. That is exactly what the discharge condition S3 named in advance *Run:* `python3 receipts/P14_matter_sector_paper/P14_turnaround_carries_D6.py`

P14R46 `P14_scale_and_ratio` [OK]
sec:correspondence (**!! CORRECTED c54.154: THE ROW SAID 'homogeneity VERIFIED by rescaling' AND THE RECEIPT PRINTED False FOR IT** – a missing 'simultaneous=True' re-applied q -¿ lambda q inside the already-scaled dependents. The receipt's own docstring diagnosed the bug and the code kept it, because nothing asserted. **Fixed and asserted c54.154; it now returns True.**). *Computes:* the conditions solved symbolically at general N_c; the cubic residual factored; homogeneity verified by rescaling; the five hypercharges compared one by one *Bound:* ** TWO CAUGHT ERRORS RECORDED IN THE BODY: a first pass claimed the cubic anomaly SELECTS N_c = 3 (FALSE – solving an identically-zero residual returns the empty set, read as a solution list rather than as vacuity); and the homogeneity check double-scaled the dependent hypercharges and

returned a spurious False. ** AND THE STANDING BOUND: the winding/charge identification is L-65's reading, whose antecedent L-74 marks NOT SHOWN, so all of this is conditional on it *Run:* `python3 receipts/P14_matter_sector_paper/P14_scale_and_ratio.py`

P14R47 `P14_tower_is_not_kaluza_klein` [OK]
sec:chirality / sec:correspondence (**!! MARKED c54.161: THE FILE EVALUATES NOTHING** – it imports sympy, never uses it, and every integer in it is a typed table entry, so its "2 + 2 = 4 exhausts the cut" is an assertion of the author rather than an output. The added checks pin the closure arithmetic and the routes, which is all there is. THE TOWER IS A PARTIAL-WAVE DECOMPOSITION, NOT A KALUZA-KLEIN TOWER. A KK tower needs an INTERNAL space; lambda indexes the Dirac operator on the transverse Sˆ2, and that Sˆ2 is the rˆ2 dOmegaˆ2 of the FOUR-metric – it is spacetime. 2 + 2 = 4 exhausts the cut, so there is nothing to reduce on and the levels are not distinct 4D fields. ** L-88 answered NO. ** And the one genuine extra dimension, the cut-normal, cannot supply a tower either for the opposite reason: it is ONE dimension and NON-COMPACT, giving a continuum. ** AND THE RANK-3 NORMAL BUNDLE IS NOT A THIRD THREE: it splits 2 + 1, the two being the transverse Sˆ2 (partial waves) and the one being the CUT-NORMAL, which is the direction R acts along by the corpus's own R-vs-T resolution. So its summands are exactly the sector's two existing gradings and it adds no rank – eliminating the wall's normal bundle from P14's own list of bundle candidates **). *Computes:* the KK/partial-wave distinction tabulated on six criteria; the dimension count 2+2=4; the normal-bundle split against the corpus's own R-vs-T resolution *Bound:* ALL THREE ROUTES of the L-15 trichotomy are now closed – (a) and (c) by P03_transverse_space_is_round, (b) here – so the 12+3=15 agreement has NO surviving explanation inside the present structure. What survives untouched is the factor-by-factor match of the TWELVE, which is a statement about the invariant sector alone, and P14's own position that the index counts walls not partial waves *Run:* `python3 receipts/P14_matter_sector_paper/P14_tower_is_not_kaluza_klein.py`

P14R48 `P14_lambda_spectrum` [OK]
prop:wall (THE ANGULAR SPECTRUM OF THE WALL-BOUND MODE – AND IT FIXES A GRADING BUT NOT A CONTENT. AN HONEST NEGATIVE. lambda is the angular Dirac eigenvalue on the cut's two-sphere: lambda = +-1, +-2, ..., multiplicity 2|lambda|; lambda = 0 excluded BY THE CONSTRUCTION (W = 0 there, so no wall). NORMALIZABILITY: psi ˜ lˆ{-2 lambda/3}, so |psi|ˆ2 dl converges iff lambda ¡ 3/4 – hence for lambda ¿ 0 the exp(-int W) branch fails and the conjugate binds, and for lambda ¡ 0 the reverse. SO WHICH SPINOR BRANCH BINDS IS FIXED BY sign(lambda): chirality is a property of the ANGULAR problem, and the mode carries TWO gradings off one number – chirality = sign(lambda) [Z_2], triality = -lambda mod 3 [Z_3]. BUT THE TOWER DOES NOT TERMINATE: every class is infinitely occupied, and the leaf's compactness gives a DISCRETE spectrum, which is not a finite one. SO THE 12+3 COUNT IS NOT DELIVERED BY THE MODE SPECTRUM – the leg counting counts PLACES, the mode counting counts STATES, and a prior reading slid between them). *Computes:* the spectrum tabulated with multiplicities; the normalizability exponent exact; the graded tally computed *Bound:* A NEGATIVE, AND ITS SCOPE IS STATED: the seat count is DOWNGRADED, not withdrawn – a seat carries a whole tower, so the reading owes a principle selecting one rung per seat and does not have one. Three live ways out are stated and none chosen. What stands untouched: the trichotomy, wall = graze point = branch point, the winding law, triality = -lambda mod 3, chirality = sign(lambda), confinement as failure to close. AND P14 ALREADY SAID THIS ('the index counts walls, NOT partial waves') – the paper drew its boundary correctly and a later reading crossed it *Run:* `python3 receipts/P14_matter_sector_paper/P14_lambda_spectrum.py`

P14R49 `P14_mode_monodromy_at_the_wall` [OK]
prop:wall / sec:chirality (THE WALL IS THE BRANCH POINT (5/5 on the five-test criterion: LOCUS, TYPE, DEFINITION, EQUIVARIANCE, SEPARABILITY – one locus with two properties, not two objects), AND THE WALL-BOUND MODE'S SHEET ADVANCE, COMPUTED. NEAR-WALL BRANCHING derived from f, not assumed: f -¿ -2M/r as r -¿ 0, so the leaf measure dl = dr/sqrt|f| gives l = sqrt2 rˆ{3/2}/(3 sqrt M), hence rˆ3 = (9/2) M lˆ2; and the turnaround law rˆ3 = 2M aˆ2 sinhˆ2(3 tau/2a) expands to rˆ3 = (9/2) M tauˆ2 – TWO INDEPENDENT ROUTES, ONE EXPONENT, so the cube-root branch point sits exactly where the wall is. MONODROMY:

r ˜ l^{2/3}, so a full LOOP gives r -¿ omega^2 r and a CROSSING (half-loop) gives r -¿ omega r EXACTLY; one crossing advances the sheet by one, three return it. THE MODE: W = lambda sqrt(f)/r and psi = exp(-int W dx) chi_+ with x the LEAF's proper distance, so W dl = lambda dr/r – the sqrt(f) CANCELS EXACTLY – giving int W dl = lambda log r and psi ˜ r^{-lambda}, a pure power of the signed areal radius. Under r -¿ omega r the mode picks up omega^{-lambda} per wall crossing, and omega^{-3 lambda} = 1 identically, so the Z_3 label is well defined on every mode of the tower. HENCE triality(mode) = -lambda mod 3: lambda = 0 mod 3 is triality zero (colourless, the lepton class), lambda != 0 mod 3 the two coloured classes, and a composite is admissible iff sum(-lambda_i) = 0 mod 3). *Computes:* the criterion tabulated; the near-wall limit, the inversion and the series exact in sympy; the cancellation W dl = lambda dr/r exact; the phases enumerated over a range of lambda *Bound:* THE SPECTRUM OF lambda IS NOT COMPUTED – which angular eigenvalues the wall problem admits on this leaf, with what multiplicity. So this fixes a GRADING and not a CONTENT: it does not say how many modes sit in each class and therefore does NOT by itself deliver the 12 + 3 count (L-81). And the method note: the five-test criterion was built at c54.24, run on ONE pair, and not swept – which is why this identity was missed for a turn (L-77) *Run:* `python3 receipts/P14_matter_sector_paper/P14_mode_monodromy_at_the_wall.py`

P14R50 `P14_dimension_from_flavour` [OK]

sec:scope (what the flavour skeleton fixes about the spacetime dimension: (1) THE COLLAPSE – the horizon relation linearises to a single multiple-angle only at D=4 (a 3-fold) and D=5 (a 4-fold); the two powers are D-3 and D-1, so the harmonics below the top number one at D=4,5 and two-or-more from D=6, against one available scale. (2) THE PARITY – 2M = r0^(D-3) - r0^(D-1) is odd in the signed offset only for EVEN D, so at D=5 the offset parity fixes each geometry, giving no mass-reflection Z2, no 3+3bar hexad and no gamma^5. =¿ D=5 would carry FOUR generations and NO chirality; four dimensions is the only one carrying both. (3) THE CHART – P3 prop:gnomonic's primary straight-line argument is dimension-independent, verified on S^n for n=2..8 (gnomonic linear to 1e-15, orthographic and stereographic not); the celestial sphere in D dimensions is S^(D-2).). *Computes:* CALIBRATED FIRST: at D=4 the collapse test returns P3's own c=2/sqrt3 and 2M=(2/(3 sqrt3)) sin 3w identically, before being trusted at other D *Bound:* assumes the D-dimensional f is the standard Tangherlini-de Sitter one – an extension of the codimension-one operator, not a rederivation of it; stated in the paper. P3's SECONDARY exclusion of the orthographic (sin theta ¡= 1 ¡ 2/sqrt3) is D=4-specific and is NOT used *Run:* `python3 receipts/P14_matter_sector_paper/P14_dimension_from_flavour.py`

P14R51 `P14_the_flat_bundle_cannot_carry_a_force` [OK]

sec:chirality (**THE FLAT BUNDLE CANNOT CARRY A FORCE, AND THE OBSTRUCTION IS AN INVARIANT RATHER THAN A GAP.** P14 asserts *'the bundle above is flat, so the construction supplies colour's exact selection rules and no force'*; this shows it. **(i) THE HOLONOMY GROUP IS FINITE, of order 81** – the three wall monodromies diag(w,1,1) and partners, together with the hinge three-cycle – and necessarily so, since the holonomy is BRANCHING and a branch structure has finitely many sheets; a finite group in characteristic zero has vanishing H^1 with any coefficients, computed here at 0. **(ii) THE MODULI OF FLAT CONNECTIONS WITH THE WALL CLASSES FIXED IS ZERO-DIMENSIONAL** – a wall branches ONE vantage and leaves the other two alone, so its monodromy has a repeated eigenvalue and its conjugacy class is SUBREGULAR, of dimension 4 rather than the regular 6; three regular classes on a three-punctured base would give 3x6 - 2x8 = 2, a two-parameter family, and the DISJOINTNESS OF THE VANTAGES' SUPPORTS is exactly what removes it. **(iii) AND THE DIMENSION WAS NEVER THE QUESTION: the moduli space of FLAT connections consists of FLAT connections, so a deformation within it changes WHICH flat bundle one has and not whether there is a field strength.** Curvature means leaving the flat locus, and no holonomy datum can, because holonomy is the complete invariant a flat connection has. **The same verdict the WINDING received one level up ('P14_quantisation_not_strength') and for the same reason: a topological datum quantises and does not measure.**). *Computes:* the four stated properties of the wall monodromies reproduced from the branch structure and asserted; the group order asserted 81 on EXACT monomial arithmetic; the moduli dimension asserted 0; the subregular class dimension asserted 4 *Bound:* **bound: this settles the HOLONOMY route only. The isometry route is excluded separately (su(3) not in so(5,1), four ways). A mechanism that is NEITHER holonomy nor isometry is not excluded here – and none has been named. The forty-one-decade scale statement in the companion computation is an argument from the substrate's single invariant, not a theorem** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_flat_bundle_cannot_carry_a_force.py`

`U2_the_matter_sector_spends_none` [OK]
sec:correspondence (the built matter sector spends zero free dimensionless constants). *Computes:* asserts P14 three no-s at source: index not datum, nothing fitted, a coupling a single length cannot build (11) *Bound:* NOT-A-PAPER-CLAIM — discharges L-200 section 1 *Run:* `python3 receipts/L200_free_data_count/`

P13R15 `L212_the_generation_index_fixes_the_mass` [OK]
sec:whichthree (the generation index fixes the mass parameter, so it cannot grade it). *Computes:* the deck shift leaves r^(D-1) and hence 2M invariant; 2M is neither invariant nor equivariant in the root (6) *Bound:* NOT-A-PAPER-CLAIM — discharges L-212 *Run:* `python3 receipts/P13_boundary/L212_the_generation_`

P14R52 `P14_the_between_horizon_leaf_converges_at_nariai` [OK]
sec:count (the between-horizon leaf length CONVERGES to pi*alpha/sqrt3 at the Nariai member; the divergent integral is over a different domain). *Computes:* asserts the double root, the sign of f on both sides, monotone convergence and the closed-form limit, analytically and numerically (12 checks) *Bound:* NOT-A-PAPER-CLAIM — withdraws routed item 24 *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_between_horizon_leaf_converges_at_nariai.py`

`C1_the_rows_premises_are_stale_and_the_split_is_specified` [OK]
sec:count (L-221 (PO-5) narrowed not closed: two of its three premises are stale, and P14 already carries a specified split, a particle-level baryon and a colourless count of four). *Computes:* asserts the corpus counts, the exterior-cube baryon, the D_6 grading count and P14s own restraint at source (15 checks) *Bound:* NOT-A-PAPER-CLAIM — a bounded narrowing of a PROTECTED row; PO-5 stays open *Run:* `python3 receipts/L221_quark_lepton/C1_the_rows_premises_are_stale_and_the_split_is_specified`

`F1_what_a_curvature_source_would_have_to_be` [OK]
sec:count (what a curvature source would have to BE on the flat selection bundle: a spatially varying m(r), because the branch points move with M and a homogeneous leaf has one M). *Computes:* asserts P14s flatness statement, computes the root motion with M, and stops at the rows own instruction (14 checks) *Bound:* NOT-A-PAPER-CLAIM — L-233 first step only; P14s negative half stands *Run:* `python3 receipts/L233_colour_curvature/F1_what_a_curvature_source_would_have_to_be.py`

`F2_the_flatness_is_what_a_branching_IS` [OK]
sec:count (the flatness is what it MEANS for the module to be a branching: the base is the 2M-plane, the fibre is the three roots, and a covering map is flat by definition). *Computes:* asserts the one-2M/three-root structure at source, the cubic and its discriminant, and withdraws F1s leaf-homogeneity claim (14 checks) *Bound:* NOT-A-PAPER-CLAIM — corrects this lines own r2467 and strengthens P14s negative *Run:* `python3 receipts/L233_colour_curvature/F2_the_flatness_is_what_a_branching_IS.py`

`C2_gradings_become_fields_by_being_a_kernel` [OK]
sec:count (a count of GRADINGS becomes a count of FIELDS by being the KERNEL of an operator, so what P13 calls unbuilt is an operator and not a sector). *Computes:* asserts P14s second-quantisation licence and index statement, the D_6 grading count, and P13s unbuilt clause at source (12 checks) *Bound:* NOT-A-PAPER-CLAIM — a narrowing of PO-5; nothing built, PO-5 stays open *Run:* `python3 receipts/L221_quark_lepton/C2_gradings_become_fields_by_being_a_kernel.py`

`M2_station_H_is_a_declared_gap_and_C_is_its_prerequisite` [OK]
sec:count (station (H) is a DECLARED traced-weight gap: the plain Atiyah-Singer theorem is about a smooth manifold and the object is a branched bead, so (C) (which group acts) is a prerequisite). *Computes:* asserts P14s own traced-weight declaration, the absence of equivariant/orbifold vocabulary, and the S_3 it names against r2491s three candidates (12 checks) *Bound:* NOT-A-PAPER-CLAIM — a reach-station result; no defect asserted *Run:* `python3 receipts/L203_reach_stations/M2_station_H_is_a_declared`

`Q1_the_missing_operator_and_the_higgs_identification_are_one_gap` [OK]
sec:count (PO-5s missing operator and L-242s undeveloped identification are ONE gap: P14 computes on Rs EVEN (massless) sector and the Higgs mechanism in CRs own terms is what takes you

off it). *Computes:* asserts P14s two-part opening, its massless-side calculation and its own external-to-the-geometry boundary, against P6s R-symmetric-sector identification (13 checks) *Bound:* NOT-A-PAPER-CLAIM — gives PO-5s vein a direction; nothing built *Run:* `python3 receipts/L221_quark_lepton/Q1_the_m`

`Q2_the_kernel_route_is_unavailable_off_the_R_even_sector` [OK]
sec:count (the R-odd direction has an exact obstruction: a mass term commutes with gamma^5 so D+m breaks the grading, and kernel-of-a-graded-operator is a structure only the R-EVEN sector has). *Computes:* builds the chiral gammas, verifies the anticommutator vanishes without the mass and not with it, and asserts P14s index machinery and its own external-to-the-geometry boundary (12 checks) *Bound:* NOT-A-PAPER-CLAIM — inverts PO-5s question; PO-5 stays open *Run:* `python3 receipts/L221_quark_lepton/Q2_the_kernel_route_is_unavailable_off_the_R_even_sector.py`

`P11_CR_fixes_the_place_not_the_couplings` [OK]
sec:correspondence (L-803 narrowed: CR fixes the right-handed neutrinos PLACE and not its couplings, N_eff counts thermalized species, so CR makes no N_eff prediction and the standard value is consistent). *Computes:* counts the gauge wall six times, asserts P0s non-claim on gauge content, shows sterile at zero, and checks that the corpus own BBN network computes the neutrino term from a decoupling temperature (12 checks) *Bound:* NOT-A-PAPER-CLAIM — changes which paragraph is owed, not whether one is *Run:* `python3 receipts/L204_physics_reach/P11_CR_fixes_the_place_not_the_couplings.py`

`B1_the_index_argument_stops_where_a_mod_two_index_begins` [OK]
sec:chirality (A6 answered: the corpus runs Atiyah–Singer on R, finds it inapplicable because the discrete orientation parity has no circle action — and never names the MOD-2 index that replaces it in exactly that case). *Computes:* asserts the per-wall anomaly use, the connected-group index argument and its stopping point verbatim, against six zero-counts for the Z2 invariants (14 checks) *Bound:* NOT-A-PAPER-CLAIM — a name and a place to look; no mod-2 index computed or claimed to exist *Run:* `python3 receipts/L221_the_bridge/B1_the_index_argument_stops_where_a_mod_two_index_begins.py`

`S3_the_wall_index_is_real_because_sqrt_f_factors_out_of_the_operator_not_the_norm` [OK]
prop:wall (#571 (joint, PO-11), the exponent half: the near-wall zero-mode index is REAL $\pm\lambda$, and the reason is that $\sqrt{}$f is a COMMON FACTOR of the operator equation ($\sqrt{}$f d/dr
ensuremath
mp W)ψ=0 with W=$\lambda\sqrt{}$f/r — so it never has to be continued through f=0 as a branch. By direct substitution r^{+λ} solves it IDENTICALLY for symbolic f (any branch), while r^{$\pm$iλ} does NOT (residual λ(i$-$1)$\sqrt{}$f r^{i$\lambda-$1}$\neq$0). The recurring i λ/r is $\int$W(1/$\sqrt{}$(abs f)): the NORM measure put where the operator's own 1/$\sqrt{}$f belongs — the transcription error shared by L-829 S1 check-2 and the four reductions (r2819). P14 keeps the two $\sqrt{}$'s separate by hand (exponent in dx=dr/$\sqrt{}$f line 193-5, norm in dl=dr/$\sqrt{}$(abs f) line 214), so its treatment is EXPLICIT — 56's r2819 dichotomy $\to$ a transcription problem, not a paper gap. CORRECTS L-829 S1 check-2 and advances B62 (the "branch choice through f=0" is a non-issue for the exponent)). *Computes:* operator-equation residuals for r^{$\pm\lambda$} vs r^{$\pm$iλ} (symbolic, exact, symbolic f), the operator integrand λ/r vs norm integrand iλ/r at f¡0, and a numerical solve of the $\sqrt{}$f-carrying complex ODE across the wall (Re index$\to+\lambda$, Im$\to$0, $\sqrt{}$f never precancelled) (6 checks) *Bound:* NOT-A-PAPER-CLAIM — the exponent half of #571 under the corpus's analytic-continuation prescription; NOT the $\omega\neq$0 continuum (56's ω-coupling $\sqrt{}$f), NOT the sqrt(abs f) self-adjoint operator choice (the observer line's), NOT #571 closed (56's gate); F5 untouched *Run:* `python3 receipts/L829_po11_continuum_continues/S3_the_wall_index_is_real_because_sqrt_f_factors_out_of`

`P14_the_three_bridges_off_the_grading_all_land_on_the_same_ceiling` [OK]
sec:whichthree (**'PO-5'/'A3'·'A4'·'A5': THE THREE BRIDGES OFF THE GRADING ALL LAND ON DIMENSION 2** — cohomology IS the kernel on a two-term grading; \$R\$ grades rather than exchanges so branching returns a character; and the only canonical spectral rung has multiplicity 2. Lead 'L-534'; VEIN 'L-221'). *Computes:* builds explicit gammas and shows \$D(p)^2=p^2\{}cdot\{}mathbb 1\{}ne0\$ so no odd nilpotent exists; SOLVES \$S_3\$'s class equation for dims 1,1,2; measures the angular gaps UNIFORM and the lowest admissible rung's multiplicity at 2; seven premise checks; and two VEIN checks that 'PO-5' is not declared closed *Bound:* VEIN MAPPED NOT CLOSED — the fourth candidate (the ANOMALY route, 'A6') is untested and nothing here bears on it; 'A4''s survival removes an owing and supplies no content *Run:* `python3 receipts/L221_quark_lepton/P14_the_three_bridges_off_the_gra`

`B4_the_radial_operator_carries_a_real_structure` [OK]

sec:wall (PO-5s mod-2 route is OPEN: the RADIAL operator carries a REAL structure — $A=\sigma z \circ \mathrm{conj}$ is the only antilinear symmetry of $H=-i\ \sigma x\ \partial\ +\ m\ \sigma z$, squares to +1, and preserves the $\sigma y=+1$ eigenspace the zero mode sits in). *Computes:* asserts P14s operator, eigenspinor and superpotential verbatim, tests all four antilinear candidates for commutation, and computes the square and the eigenspace action (14 checks) *Bound:* NOT-A-PAPER-CLAIM — establishes the condition; the index value is untouched *Run:* `python3 receipts/L221_the_bridge/B4_the_radial_operator_carries_a_real_structure`

`B5_the_mod_two_index_is_one_and_the_corpus_computed_it` [OK]

sec:wall (the mod-2 index has value 1, and P14 computed it without naming it: the orbit of (+,+,+) under the pair-flip holonomy is four of eight states with n_+ mod 2 = 1 throughout). *Computes:* asserts P14s count, holonomy action and parity-class conclusion verbatim, enumerates the orbit, and confirms the two orbits are the two parity classes (14 checks) *Bound:* NOT-A-PAPER-CLAIM — names an invariant the corpus states unnamed; the integer index stays traced *Run:* `python3 receipts/L221_the_bridge/B5_the_mod_two_index_is_one_and_the_corpus_computed_it.py`

`B6_the_object_is_delivered_and_the_row_argues_elsewhere` [OK]

abstract · sec:whichthree (PO-5s OBJECT is substantially delivered — baryon 1, diquark 0, meson 1, and the three wall monodromies with the hinge 3-cycle generate SU(3) — while its row carries a negative verdict on a DIFFERENT branching). *Computes:* asserts PO-5s object and target from the register, P14s delivered/not-delivered list verbatim from both abstract and body, and confirms SU(3) is absent from the row (12 checks) *Bound:* NOT-A-PAPER-CLAIM — narrows a register row; the coupling stays open and F5 forbids closure *Run:* `python3 receipts/L221_the_bridge/B6_the_object_is_delivered_and_the_`

`B8_po4_targets_have_opposite_verdicts` [OK]

sec:whichthree (PO-4s two targets have OPPOSITE verdicts: SU(3) is GENERATED by the wall monodromies with the hinge 3-cycle, while su(2)_L as a gauging is explicitly not delivered — and the rows status asks a reduction question the corpus never used). *Computes:* asserts the rows target and status wording, P14s generation sentence, the horn-swap clause and the two-items-not-three reduction verbatim (8 checks) *Bound:* NOT-A-PAPER-CLAIM — narrows a register row to one target with a named obstruction *Run:* `python3 receipts/L221_the_bridge/B8_po4_targets_have_opposite_verdicts.py`

`B9_po3_why_is_answered_dimensionally` [OK]

sec:whichthree (PO-3s WHY is answered dimensionally: the mass function is odd in the signed offset exactly when D is even, so at D=4 the orientation parity EXCHANGES each geometry with its conjugate — and that exchange IS the 3/3bar doubling). *Computes:* asserts the rows question and target, P13s A_2-weights identity, P14s parity clause and its consequences at D=5, and its use as a D-selection condition (8 checks) *Bound:* NOT-A-PAPER-CLAIM — connects a never-worked row to the passage that answers it *Run:* `python3 receipts/L221_the_bridge/B9_po3_why_is_answered_dimensionally.py`

`B10_po2_resemblance_is_now_a_construction` [OK]

sec:whichthree (PO-2s resemblance is now a construction: the same epsilon the row used to REMOVE a leg is the one P14 uses to BUILD the baryon — three modes one kernel, channel counts computed not matched). *Computes:* asserts the rows own epsilon clause and resemblance wording, P14s second-quantisation sentence and channel counts, P13s A_2 identity, and the flat-holonomy limit (8 checks) *Bound:* NOT-A-PAPER-CLAIM — narrows the last never-worked row; the identification stays unclaimed *Run:* `python3 receipts/L221_the_bridge/B10_po2_resemblance_is_now_a_construction.py`

`B13_po2_identification_is_stated` [OK]

sec:whichthree (PO-2s identification IS stated: each hinge designates one of the three roots of the horizon cubic, one wall per hinge, one mode per wall — and the Weyl S_3 IS the relation among the hinges, not a second group resembling it). *Computes:* asserts each link of the chain verbatim — roots to hinges, hinges to walls, walls to modes, and the same-S_3 clause — and that the row never carries it (7 checks) *Bound:* NOT-A-PAPER-CLAIM — corrects r2631s closing line; the physical identification stays unclaimed *Run:* `python3 receipts/L221_the_bridge/B13_po2_identification_is_stated.py`

`B14_identical_in_content_is_po2s_reason` [OK]

sec:whichthree (PO-2s REASON: every root returns the same 2M, so the three are identical in content and distinguished only by which root each takes as its hole — and the hinge S_3 is a within-state index, not a family symmetry). *Computes:* asserts the identical-in-content sentence, the within-state clause, the generation-wall withdrawal and the count-and-chirality clause verbatim, and that no register carries any of them (9 checks) *Bound:* NOT-A-PAPER-CLAIM — supplies a rows reason; the physical identification stays unclaimed *Run:* `python3 receipts/L221_the_bridge/B14_identical_in_content_is_po2s_reason.py`

`B15_po3_both_clauses_answered` [OK]
sec:whichthree (PO-3 is answered in BOTH clauses — the WHY by the mass functions parity in the signed offset, and the BRIDGE by the wall monodromies generating SU(3) — and both were already in its own row). *Computes:* asserts the two-clause object, P14s parity sentence, P14s generation sentence, and that the row carries both (8 checks) *Bound:* NOT-A-PAPER-CLAIM — marks a row answered; the strike is reserved *Run:* `python3 receipts/L221_the_bridge/B15_po3_both_clauses_answered.py`

`B16_the_coupling_cannot_come_from_this_bundle` [OK]
sec:whichthree (PO-5s coupling CANNOT come from the colour bundle: the branch-point holonomy is non-trivial while F vanishes identically, and force needs curvature — the same flatness that makes the selection rules exact makes a force impossible). *Computes:* asserts the flat-holonomy, quantises-not-couples and branch-point-holonomy clauses verbatim, computes the holonomy at c=1/3 and 2/3 and the vanishing curvature (8 checks) *Bound:* NOT-A-PAPER-CLAIM — a determinate negative; where a coupling WOULD come from is untouched *Run:* `python3 receipts/L221_the_bridge/B16_the_coupling_cannot_com`

`B17_the_paper_already_walls_two_routes` [OK]
sec:whichthree (P14 already walls TWO routes to a coupling — holonomy and isometry — with a dimensional argument (41.2 decades below the strong scale, verified) and names the residue: no third mechanism has been named). *Computes:* asserts the moduli-space, complete-invariant, Yang-Mills-coupling, forty-one-decades and no-third-mechanism clauses verbatim, and recomputes the decade count (7 checks) *Bound:* NOT-A-PAPER-CLAIM — corrects r2666s turn classification; the third-mechanism question stays open *Run:* `python3 receipts/L221_the_bridge/B17_the_paper_already_walls_two_rout`

`B18_the_obstruction_is_horizon_located` [OK]
sec:chirality (PO-11s obstruction is HORIZON-located, not singularity-located: the leaf and tortoise measures differ only where f vanishes, and r=0 is integrable in BOTH — so the descent is the ordinary scattering problem). *Computes:* asserts the leaf-norm and spacetime-norm clauses verbatim, and integrates both measures at a horizon and at r=0 (7 checks) *Bound:* NOT-A-PAPER-CLAIM — locates an obstruction; the scattering problem is not attempted *Run:* `python3 receipts/L221_the_bridge/B18_the_obstructio`

`B19_the_swap_is_the_weyl_z2_not_the_centre` [OK]
sec:whichthree (PO-4s remaining gap characterised: a horn swap has the WEYL Z_2s adjoint action, not the CENTREs — so what is missing is the TORUS, not the group). *Computes:* asserts the horn-swap and right-handed-pair clauses verbatim, and computes both Z_2 embeddings adjoint actions on sigma_z and sigma_x (7 checks) *Bound:* NOT-A-PAPER-CLAIM — characterises a gap; whether P14s T realises the Weyl element is not read here *Run:* `python3 receipts/L221_the_bridge/B19_the_swap_is_the_weyl_z2`

`B20_centre_and_weyl_are_opposite_ends` [OK]
sec:whichthree (THE TWO GAUGE DEBTS SIT AT OPPOSITE ENDS OF THE SUBGROUP LATTICE: colour arrives at the CENTRE (adjoint-trivial, a label) and isospin at the WEYL element (adjoint-non-trivial, a reflection) — so PO-5 is excluded and PO-4 is one factor short). *Computes:* asserts P14s unbanked three-routes statement verbatim, verifies the SU(3) centre is central against non-commuting generators and adjoint-trivial, and the SU(2) Weyl element is not (7 checks) *Bound:* NOT-A-PAPER-CLAIM — connects two rows; the three colour routes are P14s and are used, not reproved *Run:* `python3 receipts/L221_the_bridge/B20_centre_and_weyl_are_opposite_ends.py`

`B22_the_tower_is_uniform` [OK]
sec:chirality (PO-11s obstruction is UNIFORM IN THE TOWER: run on the actual modes between the true horizons, the leaf norm is finite and cut-off independent at every lambda while the tortoise norm diverges logarithmically — no high-j corner rescues it). *Computes:* asserts the tower and single-branch clauses verbatim, checks the horizons are cubic roots, and runs both norms at

lambda = 1, 3, 10 across three cut-offs with a logarithmic-growth test (7 checks) *Bound:* NOT-A-PAPER-CLAIM — confirms an obstruction; the scattering problem is untouched *Run:* `python3 receipts/L221_the_bridge/B22_the_tower_is_uniform.py`

`B24_the_triality_test_run` [OK]
sec:whichthree (THE TRIALITY TEST RUN: computed from the COLOUR CONTENT alone via the centres Z_3 grading, t=(p-q) mod 3 gives meson 0, baryon 0, diquark 2 — exactly P14s own "baryon 1, diquark 0, meson 1", with no charge on either side). *Computes:* asserts the owed-test and circularity clauses verbatim, computes the triality of four channels, and matches the count against P14s own sentence (8 checks) *Bound:* NOT-A-PAPER-CLAIM — the SELECTION agrees; the derived-vs-assumed identification is untouched, and the paper edit is routed *Run:* `python3 receipts/L221_the_bridge/B24_the_triality_test_run.py`

`B25_the_scattering_object_exists` [OK]
prop:wall (PO-11s missing OBJECT is one line from the corpus own superpotential: W = lambda sqrt(f)/r gives Regge-Wheeler partners V_pm = W^2 +- dW/dx that VANISH AT BOTH HORIZONS, so scattering states with continuum normalisation are DEFINED). *Computes:* asserts B3s superpotential and vanishing-at-every-horizon clauses verbatim, computes both partners at both horizons and mid-region, and confirms Regge-Wheeler appears nowhere in the papers (6 checks) *Bound:* NOT-A-PAPER-CLAIM — the SUSY-QM partner form is standard and applied not proved; completeness is untouched *Run:* `python3 receipts/L221_the_bridge/B25_the_scattering_object_exists.py`

`B26_the_spectrum_computed` [OK]
prop:wall (PO-11s SPECTRUM COMPUTED: the barrier from B3s superpotential solved on the tortoise line — transmission 0.020 to 0.9997 across two decades, T=0.691 at the barrier top, and UNITARITY to six figures). *Computes:* locates the barrier peak, shows the tortoise width diverges logarithmically, integrates the scattering problem at five energies and checks abs(A)^2-abs(B)^2 = 1 throughout (5 checks) *Bound:* NOT-A-PAPER-CLAIM — one parameter value, V_- not run, and COMPLETENESS is what remains *Run:* `python3 receipts/L221_the_bridge/B26_the_spectrum_computed.py`

`B27_completeness_follows` [OK]
prop:wall (PO-11 CLOSES: completeness FOLLOWS rather than remaining — V_+ is bounded (sup 1.963) and exponentially decaying in x, which is Kato-Rellich plus the short-range condition, so bound tower plus continuum is complete). *Computes:* measures the sup over 20,000 points, shows a constant decay factor per unit tortoise length at two successive scales, and confirms the decay rate is scale-independent (5 checks) *Bound:* NOT-A-PAPER-CLAIM — Kato-Rellich and the spectral theorem are cited not re-derived; angular/flavour structure and coupling are untouched *Run:* `python3 receipts/L221_the_bridge/B27_completeness_follows.py`

`B28_a_cardinality_gap` [OK]
sec:whichthree (PO-4s gap is a CARDINALITY gap: the reflection generates a group of order 4 while the maximal torus is a continuum, so no product of reflections reaches it — and PO-5s wall does NOT transfer, since it turns on connection data). *Computes:* computes the reflections group order and the tori distinctness, checks exponential-map and one-parameter-subgroup at zero while root-system/Weyl/Lie-algebra are present, and pins PO-5s wall clause (8 checks) *Bound:* NOT-A-PAPER-CLAIM — the cardinality argument fells one route, not every route; PO-4 is NOT excluded *Run:* `python3 receipts/L221_the_bridge/B28_a_cardinality_gap.py`

`B29_the_two_walls_are_one` [OK]
sec:whichthree (PO-5 IS NOT UNBOUNDED: the two walls are ONE wall — a coupling is the coefficient of an F^2 term, so where F=0 the dimensional argument is moot — and the question restates as WHERE CAN A GAUGE CONNECTION HERE FAIL TO BE FLAT). *Computes:* quotes both walls verbatim, shows the flatness claim is SCOPED (the bundle ABOVE), and finds the word UNBOUNDED lives only in this lines own stamp script and never in the row (7 checks) *Bound:* NOT-A-PAPER-CLAIM — no mechanism supplied; neither wall weakened on its own ground *Run:* `python3 receipts/L221_the_bridge/B29_the_two_walls_are_one.py`

B30_the_parameter_has_the_wrong_signature [OK]

sec:whichthree (PO-4s ONE-PARAMETER SUBGROUP SEARCHED: the horn angle fails as Z_3 (colours centre, not SU(2)s Cartan); P14 locates the doublet as the two TIMELIKE-separated ends of one hinge; and a rapidity is continuous and diagonal but NOT UNITARY — SL(2,R), not SU(2)). *Computes:* tests the horn rotation over the circle, quotes P14s timelike-ends clause verbatim, and shows the rapidity family is continuous, diagonal, det 1 and non-unitary (7 checks) *Bound:* NOT-A-PAPER-CLAIM — the enumeration is over what the substrate supplies; a construction not yet built could supply another *Run:* `python3 receipts/L221_the_bridge/B30_the_parameter_has_the_wrong_signature.py`

B31_the_wall_reason_is_too_narrow [OK]

sec:whichthree (THE HOLONOMY WALL HOLDS AND ITS STATED REASON IS TOO NARROW: "only phases" is a fact about COMPACT groups, and r2733 found the corpus own candidate generates SL(2,R), where an n-fold loops holonomy runs 1.65 -¿ 148). *Computes:* computes the holonomy family against winding, contrasts the compact cases, and pins the rows own record of the flat-connection argument (5 checks) *Bound:* NOT-A-PAPER-CLAIM — the wall stands on F=0; one of two reasons offered for it is withdrawn *Run:* `python3 receipts/L221_the_bridge/B31_the_wall_reason_is_too_narrow.`

C1_the_tortoise_divergence_is_the_normalisation_of_the_object_not_the_obstruction_to_it [OK]

sec:chirality (PO-11's OBSTRUCTION IS A NORMALISATION CONDITION READ AS A BARRIER: the tortoise norm diverges "where the mode tends to a constant" – which is a plane wave – and the omega != 0 continuum, never posed, is BUILT here). *Computes:* the SUSY-QM partner pair V_pm = W^2 +/- dW/dr_* of the massless radial Dirac operator on P14's undercritical SdS: W's exponential decay measured against both surface gravities (1.56031 vs 1.5603127, -0.67895 vs 0.6789488); the scattering solutions integrated and |T|^2+|R|^2 = 1 to 1e-11 at six (lambda, omega); the SUSY partners isospectral to 8 digits; the continuum state's tortoise integral growing linearly with slope |A|^2+|B|^2 = 7.83140 (measured 7.83140); and L^2(tortoise) contained in L^2(leaf) since sup sqrt(f) = 0.5197459 is bounded. CONTROLS: a seeded non-decaying tail breaks the asymptotic extraction; the leaf norm of the SCATTERING state is finite too; and both surface gravities go to zero as M -¿ Nariai *Bound:* not the full propagating sector – the radial continuum at fixed (lambda, wall) in the static region, which is the row's LAST SENTENCE and not the whole row; not a greybody spectrum; not that CR should read the spacetime norm; nothing at a double root *Run:* `python3 receipts/L548_propagating_sector/C1_the_tortoise_divergence_is_the_normalisation_of_the_object_not_the`

M1_the_third_mechanism_is_bounded_and_the_bound_is_two_sentences_earlier [OK]

sec:whichthree (PO-5's RESIDUE IS BOUNDED, AND THE BOUND IS TWO SENTENCES EARLIER: P14's dimensional sentence mentions no route, so it constrains the TARGET and bounds a THIRD mechanism exactly as it bounds the first two – and its premise is p0's ledger position, so the coupling question and the constant-ledger question are one). *Computes:* the residue and the bounding sentence located in P14 and their separation measured (two sentence-ends, same paragraph); the bounding sentence tested for route-words (holonomy, isometry, flat, bundle, monodromy, winding – none present); p0's ledger sentence and its cross-register reason quoted; and the arithmetic reproduced across the observed Lambda range – alpha = sqrt(3/Lambda), hbar c / alpha at 41.2 decades below the strong scale, alpha/l_P = 10^61.0, Lambda l_P^2 = 2.9e-122. Dimensional counting [g^2] = L^(D-4) gives the wall at D=4 only and a LENGTH at D=5. CONTROL: the same counting on Einstein-Hilbert gives [1/16 pi G] = L^(2-D), dimensionful in every dimension, so gravity is the case the argument does not touch and it does not prove too much *Bound:* not that no third mechanism exists – this bounds the search and F5 forbids closing it; not that l_P is a gauge, which is p0's stated position quoted and depended on rather than established here; not a five-dimensional gauge sector – the scope claim says where the wall stops, not that anything stands past it; and not that 10^61 cannot yield g^2 ~ 1 by some function, the claim being that the ledger offers no FREE parameter *Run:* `python3 receipts/L550_third_mechanism/M1_the_third_mechanism_is_bounded_and_the_bound_is_two_sentences_earlier`

W1_the_weyl_group_is_a_quotient_not_a_complement_so_the_row_is_two_dimensions_short [OK]

sec:whichthree (PO-4 IS NOT "ONE FACTOR SHORT": ¡T, Weyl¿ generates N(T), dimension 1 against SU(2)'s 3, because the Weyl group is the QUOTIENT N(T)/T and not a complement – so what is absent is the root subgroups, not the U(1)). *Computes:* the Weyl element w = i sigma_x verified to flip sigma_z and fix sigma_x with w^2 = -1 (so the horn-swap identification and the centre-against-Weyl contrast both hold); the group generated by the maximal torus and w enumerated to six word-lengths and found to reach only diagonal and antidiagonal elements, i.e. N(T); a generic SU(2) element shown unreachable; and one root subgroup shown to reach general elements at once. CONTROLS: ¡T,W¿ = G exactly when G is abelian – the case with nothing to gauge, so a control returning "true for SU(2) too" would have meant the computation tested nothing – and the codimension computed across SU(2), SU(3), SU(5) as 2, 6, 20 *Bound:* NOT-A-PAPER-CLAIM – this corrects a register row's arithmetic and asserts nothing new about the physics; not that the horn swap is not the Weyl element, which is r2676's and is verified here; not that PO-4 is harder than PO-5, only that they are not separated by SIZE as stated; the chirality mismatch (SU(2)_L acting on LEFT-handed doublets against P14's right-handed occupations) is untouched and stays the harder question; and not a closure – PO-4 stays open *Run:* `python3 receipts/L552_weyl_and_torus/W1_the_weyl_group_is_a_quotient_not_a_complement_so_the_row_is_two_dimensi`

`B32_the_two_bounds_compose` [OK]

sec:whichthree (PO-5s TWO BOUNDS COMPOSE and cc54s is the tighter half: r2729 says there is no F^2 TERM to put a coefficient in front of, c54.216 says there is no NUMBER to be the coefficient — a third mechanism must supply BOTH). *Computes:* pins P14s constrains-the-target and fixed-pure-number clauses and p0s gauge-units ledger position verbatim, and confirms the two halves are different objects (5 checks) *Bound:* NOT-A-PAPER-CLAIM — a third mechanism is not excluded; the bound says what one must deliver *Run:* `python3 receipts/L221_the_bridge/B32_the_two_bounds_compose.py`

`B34_the_join_crosses_the_inner_horizon` [OK]

prop:wall (PO-11s JOIN IS LOCATED: the wall sits at r=0, deep inside the inner horizon where f -¿ -infinity, while the continuum lives in the static region — so the remaining undertaking is a MATCHING ACROSS A KILLING HORIZON). *Computes:* quotes P14s two-kinds-of-zero and domain-wall clauses, evaluates f at five radii to place r=0 inside the inner horizon, and pins the undertaking clause (6 checks) *Bound:* NOT-A-PAPER-CLAIM — the join is not shown impossible; what is established is the KIND of problem it is *Run:* `python3 receipts/L221_the_bridge/B34_the_join_crosses_the_inner_horizon.py`

`B35_the_continuation_is_cited_wrong` [OK]

prop:wall (THE CONTINUATION PO-11s JOIN NEEDS EXISTS, and P14s join sentence cites the two papers WITHOUT it: geometric_core and boundary carry no horizon-crossing vocabulary, while janzen_circle continues C-infinity through the wall own locus r=0). *Computes:* counts the crossing vocabulary as zero in both cited papers, finds Kruskal 20x and horizon-regular elsewhere, and quotes the branch-point-not-a-barrier clause against P14s odd-in-signed-radius wall (6 checks) *Bound:* NOT-A-PAPER-CLAIM — the mode matching does not follow; what is established is that the GEOMETRIC half is done and cited wrong *Run:* `python3 receipts/L221_the_bridge/B35_the_continuation_is_cited_wrong.py`

`B36_two_of_three_crossings_are_supplied` [OK]

prop:wall (PO-11s MATCHING IS TWO-THIRDS SUPPLIED: the METRIC crosses r=0 as a theorem and the BOUND MODE crosses it in P14s own proposition — and those two are ONE structure, since P14s chirality sign IS the signed-radius flip. Only the CONTINUUM has not been asked). *Computes:* pins the continuation theorem, the zero-mode proposition and its chirality clause, and locates the continuum on the static regions tortoise line (6 checks) *Bound:* NOT-A-PAPER-CLAIM — the continuum is not shown to continue; what is established is that it is the only one of three left *Run:* `python3 receipts/L221_the_bridge/B36_two_of_three_crossings_are_supplied.py`

`B38_the_ledger_is_discrete` [OK]

sec:whichthree (r2768s MERGE WITHDRAWN — THE LEDGER IS DISCRETE, WHICH SPLITS THE ROWS: PO-5 needs a FIXED number and PO-4 needs one that RANGES, opposite properties. Every dimensionless value the corpus holds (3, 6, 3, 3/4, 9/10, 1.0824) is fixed, so PO-5 is compatible and PO-4 is not). *Computes:* pins P14s fixed-pure-number clause against PO-4s ranging requirement, and the discrete-horn-swap and F^2 records (6 checks) *Bound:* NOT-A-PAPER-CLAIM — a continuous compact parameter is not excluded; the enumeration is over what the corpus HOLDS *Run:* `python3 receipts/L221_the_bridge/B38_the_ledger_is_discrete.py`

`B40_the_isometry_reaches_colour_not_isospin` [OK]
sec:whichthree (THE SO(4) ISOMETRIES ACT ON THE ZERO-MODES AND ON THE WRONG INDEX: the leaf is compact with a well-defined Dirac index so the action is automatic, but an isometry MOVES POINTS and therefore permutes hinges — the three-space colour occupies, not the two ends within a hinge). *Computes:* pins the leaf-compactness clause, both candidate spaces (three zero-modes vs two timelike hinge ends), the triality structure and P14s own colour/isospin asymmetry (6 checks) *Bound:* NOT-A-PAPER-CLAIM — no SU(2) on the two-end space is excluded; what is shown is that the LEAF ISOMETRIES do not reach it *Run:* `python3 receipts/L221_the_bridge/B40_the_isometry_reaches_col`

`B41_the_isospin_structure_is_finite` [OK]
sec:whichthree (PO-4s KIND QUESTION RESOLVES: P14 works in D_6, which HAS two-dimensional irreps — the doublet EXISTS — but D_6 is FINITE and a gauge field is a connection valued in a Lie algebra. The doublet is not missing; the CONTINUUM acting on it is). *Computes:* pins P14s D_6 clause and its dimension-four count, verifies the irrep dimensions against the order, and pins P14s species-label statement (6 checks) *Bound:* BOUNDED NEGATIVE — the row is protected and this resolves the KIND question only; a construction embedding D_6 in a continuous group would change it *Run:* `python3 receipts/L221_the_bridge/B41_the_isospin_structure_is_finite.py`

`B43_the_two_sides_of_the_wall_differ_in_signature` [OK]
prop:wall (THE TWO SIDES OF THE WALL DIFFER IN SIGNATURE: f -¿ -inf as r -¿ 0+ (inside the inner horizon, r timelike) and f -¿ +inf as r -¿ 0- (static, r spacelike), because 2M/r is ODD and dominates at the origin — so the crossing is a SIGNATURE ROUND TRIP, not one matching). *Computes:* computes f across the origin, its real roots (a reflected de Sitter horizon on the conjugate branch), and confirms r2744s own sample stands (6 checks) *Bound:* NOT-A-PAPER-CLAIM — the round trip is the shape of the question, not a verdict; the metric continuation remains a theorem *Run:* `python3 receipts/L221_the_bridge/B43_the_two_sides_of_the_wall_differ_in_signature.py`

`B46_one_matching_not_two` [OK]
prop:wall (ONE MATCHING, NOT TWO: the inner horizon is at INFINITE tortoise distance (simple zero of f, log divergence) and the wall at r=0 is at FINITE (f ˜ -2M/r so 1/f vanishes, r_* = -0.385239 stable to 1e-12) — so r2785s two signature changes are not two junctions). *Computes:* checks f(r_b) is a simple zero, shows naive quadrature WANDERS on the log singularity rather than converging, and confirms the wall integral is stable across four cutoffs (5 checks) *Bound:* NOT-A-PAPER-CLAIM — the matching condition is not derived, only WHERE it must be imposed; the potential V is not examined *Run:* `python3 receipts/L221_the_bridge/B46_one_matching_not_two.py`

P12R17 `A8_the_self_protecting_falsehood` [OK]
sec:triality (THE SELF-PROTECTING FALSEHOOD: r2705 ran the triality test and it passed, the ledger was marked discharged, and 'check_open_ledger' fired because **P14 still SAID "this sector does not yet do so"**. The gate was read as "the ledger is wrong, restore it" when the correct read was **"THE PAPER IS WRONG, FIX IT"** — three layers, the third a systems failure. **Registered c54.225 ('L-559'): the file existed and had NO INDEX row.**). *Computes:* reconstructs the three layers from the register, the paper and the gate's own output, and names which layer each belongs to *Bound:* NOT-A-PAPER-CLAIM — a diagnosis of a process failure. **Registered, not written, at c54.225.** *Run:* `python3 receipts/P12_algebroid/A8_the_self_protecting_falsehood.py`

`B48_the_field_is_dirac_so_the_matching_is_determined` [OK]
prop:wall (THE FIELD IS DIRAC BY THE ROWS OWN OBJECT COLUMN ("a propagating DIRAC sector", Dirac 8 times, scalar 0) — so L-828s FERMION branch applies: indices +/-lambda, NON-degenerate, the condition is a POWER and the matching is DETERMINED, not chosen). *Computes:* reads the rows object column, pins L-828s non-degenerate fermion result and B47s scalar double root as the case the row does NOT ask about (5 checks) *Bound:* NOT-A-PAPER-CLAIM — the continuum is not constructed; the condition is determined and imposing it is the remaining work *Run:* `python3 receipts/L221_the_bridge/B48_the_field_is_dirac_so_the_matching_is_determined.py`

`B52_the_missing_f2_is_entailed_by_the_delivery` [OK]
sec:twofactors (THE MISSING F^2 IS ENTAILED BY THE DELIVERY, NOT AN OVERSIGHT:

P14 says colour acts on THE BRANCHING "rather than any bundle of the substrate", a branching is a covering, and a bundle associated to a pi_1-representation is FLAT by construction — so F = 0 and F^2 = 0). *Computes:* pins P14s branching, monodromy and flatness clauses, and checks that covering monodromies are unit-determinant and multiply to the identity (6 checks) *Bound:* NOT-A-PAPER-CLAIM — P14 asserts the flatness rather than deriving it; the reason here is standard bundle theory, and no alternative delivery is proposed *Run:* `python3 receipts/L221_the_bridge/B52_the_missing_f2_is_entailed`

`B54_the_retraction_supports_the_verdict` [OK]

prop:wall (cc54s RETRACTION OF r^{+/-i lambda} IS LOAD-BEARING *FOR* r2800s VERDICT: real indices separate (one decays, one grows) so normalisability picks one, while imaginary indices have unit modulus at every radius — the LIMIT-CIRCLE case, which HAS the one-parameter freedom the verdict denies). *Computes:* computes both index forms at three radii, pins P14s angular-label definition of lambda, and confirms r2800s first step is index-independent (5 checks) *Bound:* NOT-A-PAPER-CLAIM — the transmission amplitude does not follow; #556 needs the correct radial operator carried across the horizon *Run:* `python3 receipts/L221_the_bridge/B54_the_retraction_supports_the_verdic`

`B55_right_kind_right_place_numerical_declined` [OK]

sec:twofactors (1/sqrt3 IS THE RIGHT KIND IN THE RIGHT PLACE, AND THE NUMERICAL CLAIM IS DECLINED: P14 puts the obstruction AFTER the descent ([g^2] = L^(D-4), a length at D=5), so the descent must divide one geometric length by another — which is what r_N/alpha is). *Computes:* pins P14s bound and its dimensional sentence, computes the D=5/D=4 counting and r_N/alpha, and shows the alpha_s comparison is NOT clean (6 checks) *Bound:* NOT-A-PAPER-CLAIM — the structural fit is necessary not sufficient, and the numerical test is DECLINED because alpha_s runs and P14 does not name the scale *Run:* `python3 receipts/L221_the_bridge/B55_right_kind_right_pla`

`B57_the_rules_are_centre_data_the_force_is_root_vectors` [OK]

sec:twofactors (THE RULES ARE CENTRE DATA AND THE FORCE IS ROOT-VECTOR DATA: su(3) = 2 (Cartan) + 6 (root vectors); the zero-sum roots span the FULL Cartan and S_3 is the Weyl group, while baryon-1/diquark-0/meson-1 are TRIALITY — centre grading, needing no root vectors at all). *Computes:* computes the Cartan rank from the zero-sum constraint, the three triality sums, and the commutator showing the hinge 3-cycle is what makes the monodromies non-abelian (6 checks) *Bound:* NOT-A-PAPER-CLAIM — P14s "generate SU(3)" is not contradicted; this identifies WHICH part the delivered results need. The finite groups order is not load-bearing *Run:* `python3 receipts/L221_the_bridge/B57_the_rules_are_centre_data_the_force_is_root_vectors.py`

`B58_p14_already_answers_the_owed_question` [OK]

sec:twofactors (r2811s OWED QUESTION IS ANSWERED IN P14, AND A STRONGER VERSION: "the moduli space of flat connections consists of flat connections, so a deformation within it changes which flat bundle one has and not whether there is a field strength" — so a continuous direction would not help). *Computes:* pins P14s flat-connections sentence, its dimension-is-a-bonus clause, the subregular count and the finitely-many-sheets reason, and checks P14s order 81 against a closure (5 checks) *Bound:* NOT-A-PAPER-CLAIM — the owed item discharges as ANSWERED-IN-CORPUS, which shrinks the debt without advancing the row *Run:* `python3 receipts/L221_the_bridge/B58_p14_already_answers_the_owed_question.py`

`B60_one_operator_the_fork_does_not_exist` [OK]

prop:wall (ONE OPERATOR: B3s tetrad fixes the omega-coupling as omega/sqrt(f), and multiplying the frame pair by sqrt(f) returns dP/dx -/+ (lambda sqrt(f)/r) P = -/+ i omega P — Chandrasekhar, and L-813s form. There is no lambda f/r and the fork does not exist). *Computes:* pins B3s tetrad and superpotential and carries the frame reduction to the tortoise, and records that a THIRD naive reduction gives a third wrong answer for the indices (6 checks) *Bound:* NOT-A-PAPER-CLAIM — the operator is settled and P14s real +/- lambda is NOT derived from it; the f¡0 branch question is named, not resolved *Run:* `python3 receipts/L221_the_bridge/B60_one_operator_the_fork_does_not_exist.py`

`B62_the_rescaling_cannot_make_an_index_real` [OK]

prop:wall (THE RW-TO-PHYSICAL RESCALING IS f^(-1/4) AND CANNOT PRODUCE A REAL INDEX: a real prefactor shifts an exponent by a real amount and cannot change its imaginary part — so cc54s planned route to P14s +/- lambda is eliminated before the shift is spent). *Computes:* computes check-2s integrand under both measures (confirming cc54s flag against L-829 S1) and derives dx/dl = f^(-1/2) giving psi = P f^(-1/4) (5 checks) *Bound:* NOT-A-PAPER-CLAIM — one route is eliminated, not the problem; the f^(-1/4) factor is the correct measure conversion and belongs in any careful treatment *Run:* `python3 receipts/L221_the_bridge/B62_the_rescaling_cannot_make_an_index_real.py`

`B65_the_gluon_is_the_octet_and_p14_counts_the_singlet` [OK]

sec:whichthree (A COMPOSITE GLUON IS THE OCTET AND P14 COUNTS THE SINGLET: 3 (x) 3bar = 1 + 8, and "baryon 1, diquark 0, meson 1" is a colour-SINGLET count — so the octet channel where a gluon would live is outside its question, for the same reason Weinberg-Wittens confinement escape holds). *Computes:* computes the spin and colour decompositions, and pins P14s partial-wave statement and its own gloss of the count as CHANNELS rather than states (5 checks) *Bound:* NOT-A-PAPER-CLAIM — the octet channel exists by representation theory and whether the wall kernel populates it with a massless spin-1 is the unasked question; P14s count is not incomplete, the octet is outside it *Run:* `python3 receipts/L221_the_bridge/B65_the_gluon_is_the_octet_and_p14_counts_the_singlet.py`

`B67_verdict_the_analytic_sqrt_f_operator` [OK]

prop:wall (VERDICT (F5): THE ANALYTIC sqrt(f) OPERATOR. Self-adjointness is a requirement on a SPATIAL Dirac operator; where f¿0 r is spacelike and the two prescriptions AGREE, and where f¡0 r is TIMELIKE so the radial operator is an evolution generator and self-adjointness does not apply). *Computes:* pins S3s overall-factor result and classifies r as spacelike or timelike at five radii spanning the inner horizon (5 checks) *Bound:* NOT-A-PAPER-CLAIM — the omega != 0 continuum is not constructed; a sqrt(|f|) treatment is correct where r is spatial, which is where it agrees *Run:* `python3 receipts/L221_the_bridge/B67_verdict_the_analytic_sqrt_f_operator.py`

`K1_the_traced_index_is_computable_and_the_computation_is_a_limit_point_test` [OK]

R-M station (H) / index theory (**THE γ^5-GRADED INDEX P14 STATES AT *TRACED* WEIGHT IS COMPUTABLE AT THE POINT WHERE IT IS TRACED, AND THE COMPUTATION IS A WEYL LIMIT-POINT TEST.** Index theory's first question is not a theorem but **is your operator Fredholm, and on what domain** — and 'Fredholm' occurs ZERO times in seventeen papers, while the limit-point/limit-circle/deficiency-index apparatus occurs only in 'P10' and never in 'P14'. From the corpus's own objects: 'W dℓ = λ dr/r' exactly (the $\sqrt{}$f cancels for every M, α, r), so the branches are 'ψ ~ r^
ensuremath
mpλ', and near the branch point '$|\psi|^2$dℓ ~ r^(
ensuremath
mp2λ+$\frac{1}{2}$)' — **both are L^2 iff $|\lambda|$ ¡ $\frac{3}{4}$**. The attained spectrum is $\lambda = \pm1, \pm2, \dots$ with λ=0 excluded by W=0, so **no attained value lies in the window**: every mode is limit-point, the operator is essentially self-adjoint at the branch point, and **no boundary condition exists to be chosen**. Compactness in the leaf measure — which P14 establishes — makes the index FINITE; this is what makes it CANONICAL. **And λ=0 is doubly excluded by two facts that are not the same fact:** W$\equiv$0 there, and it is the only value inside the window.). *Computes:* COMPUTES: SYMBOLIC in M and α — the exact 'W dℓ = λ dr/r' cancellation and '$\int$W dℓ = λ log r', and the near-branch form of f; the L^2 threshold analytically (p ¿ $-\frac{3}{4}$) and again by **CUTOFF-SCALING** integration, whose sub-threshold half (λ=0.5, 0.70, not in the spectrum) is the control showing the test measures the exponent and not the cutoff; the attained spectrum quoted from P14 against the window; the D-generalisation of BOTH sides — window (D$-$1)/4 against lowest attained (D$-$2)/2 on the round \$S^{D−2}\$ — giving canonical for every D $\geq$ 3 with equality only at D=3 and ratio 4/3 at four; **the spurious convergence reconstructed** (holding the angular spectrum at its D=4 value returns D $\in$ {3,4,5}, and P03's window is {4,5} with the same caveat at five); and the index of a Dirac operator on six closed loops, including two with NO zeros, all returning 0 (20 checks). *Two parameters are pinned and named (M=0.12, α=1) and enter only the numerical confirmation; the analytic result is independent of both.* *Bound:* **NOT claimed:** that the Atiyah–Singer index on the bead is computed — it is not, and what is computed is the prerequisite the traced statement needed and did not have; that the count 3 is re-derived — the wall count is P14's and is untouched; that P14 or its receipt overstated anything — both marked the weight correctly, and the receipt's own head says the

Atiyah–Singer statement is traced; that the bead's topology is settled — the loop computation is a CONTROL on a smooth circle, not a model of the bead; that D=3 is excluded by this. *Run:* `python3 receipts/L264_station_H_the_index_is_canonical/K1_the_traced_index_is_computable_and_the_computation_i`

P14R54 `I51_the_monodromy_bound_is_attained_and_the_four_dimensional_contrast_is_real` [OK]
sec:count (P14 and its own receipt both stop at an UPPER BOUND on the D=5 monodromy ("confines it to the centraliser of (0 1)(2 3) in S_4, of order eight against twenty-four"). Contained in a group of order 8 is consistent with the monodromy being TRIVIAL, which would make the pair structure vacuous rather than smaller. This computes the group: order EXACTLY 8, the bound attained). *Computes:* continues the roots of r^{D-1}-r^{D-3}+t around each branch point of the mass plane and generates the permutation group; D=5 returns 8 and D=4 returns the full S_3 = 6, which is the four-dimensional statement the paper contrasts against; stable under step counts 1000-8000, three loop radii and three basepoints, and each generator verified to be a non-identity permutation *Bound:* SUPPORTS P14 and CLOSES its qualifier — the cover factors through u=r^2, so a loop about t=1/4 exchanges the pairs and one about t=0 exchanges within a pair, generating the order-8 wreath product structurally *Run:* `python3 receipts/P14_matter_sector_paper/I51_the_monodromy_bound_is_attained_and_the_f`

P14R4 `P14_the_two_threes_are_not_related_as_covers` [OK]
sec:whichthree · P7 rem:twocovers (**THE CORPUS'S TWO THREE-FOLD STRUCTURES ARE NOT RELATED AS COVERS.** The turnaround deck FIXES the root cover's base and does not permute its fibre: with r a root of r^3 - r + 2M and omega^3 = 1, (omega r)^3 - omega r + 2M = r^3 + 2M - omega r = r - omega r = r(1 - omega), **which vanishes only at r = 0**. So the two threes cannot be related by ANY covering-space construction over the 2M-plane – strictly stronger than lem:twoturnings, which rules out AFFINE identifications of the ROOT SETS and is exhibited here as the special case (a, b) = (omega, 0). Companion, at lower weight: the branch sets are {+-alpha/sqrt3} and {0} and are DISJOINT – the two loci P7 pries apart, so that naming guard has an analytic cause. **THE RESULT IS THE OBSERVER LINE'S, ROUTED AS 'FOR_54' ITEM 10; THE CHECK IS THIS LINE'S, and it exists because the routed receipts 'C1'/'C2' carry ZERO assertions between them** – 'lint_assertions' puts both among the thirteen that carry no check of any kind, and '\{}rcpt{X}' means X would FAIL if the claim were false, so neither could be cited from a paper. The mathematics was verified independently here before anything was landed. [borrowed from P14 (+P7)]). *Computes:* the identity asserted EXACT and equal to abs(r(1-omega)) = sqrt(3)*abs(r) across 54 (2M, root, omega) triples – **not merely non-zero, since a non-zero test passes on a wrong formula**; the deck asserted to fix exactly ONE root and that root asserted to be r = 0; the collided pair asserted to sit at alpha/sqrt3, so a moved branch locus fails here; and at least 20 roots asserted to leave the root variety, so the sweep cannot be empty *Bound:* **scope: this shows the two covers are not related as covers. It does NOT show they share no structure – they share r and 2M, and both count three because f = 0 has degree D-1, which is sec:whichthree's own statement and is untouched.** *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_two_threes_are_not_related_as_covers.py`

P3R32 `P03_R_reaches_the_turnaround` [OK]
sec:ellipse / sec:temporal-threeness / prop:wall (DOES R = gamma^5 REACH THE TURNAROUND THREE? The weld split's falsifier, ANSWERED IN THE POSITIVE. R is M -¿ -M; on r^(D-1) = 2M alpha^2 sinh^2(...) with real tau the right side becomes non-positive, so there is a REAL image iff D-1 is odd, iff D IS EVEN, and the image is r -¿ -r, THE SIGNED RADIUS – the corpus's own conjugate branch. AND THAT IS THE SAME CONDITION THE DIMENSION RESULT USES ON THE HORIZON SIDE (2M = r0^(D-3) - r0^(D-1) odd in the signed offset iff D even), verified to agree at D = 4,5,6,7,8. SO THE CHIRALITY GRADING SURVIVES THE WELD SPLIT VERBATIM: P14's parity argument transfers to generation's new seat without modification, including the D=5 conclusion. THE SPLIT IS SAFE. Groups: -1 is not in the deck orbit, so R is outside the deck Z_3 and the two generate Z_6; the turnaround three carries Z_6 where the horizon three carries D_6 = S_3 x Z_2, and the missing index-2 part is exactly the TRANSPOSITIONS, which the split assigns to colour where the singlet needs them [borrowed from P3 / P14]). *Computes:* the reality condition on r^(D-1) tabulated; the horizon-side parity checked symbolically; the deck-orbit membership of -1 checked *Bound:* NOT SHOWN: that the wall-bound zero mode's chirality is graded by the TURNAROUND-side R rather than the horizon-side one. The two coincide at D=4 by the agreement above so nothing

is wrong, but the identification is a further step (L-94). The P14 revision is now UNBLOCKED but not made *Run:* `python3 receipts/P03_SdS_slicing/P03_R_reaches_the_turnaround.py`

P3R33 `P03_the_weld_and_the_two_folds` [OK]
sec:temporal-threeness / prop:cosmo (THE WELD: CAN IT SPLIT, AND WHY ARE BOTH OF THE CORPUS'S THREES D-1? (answers L-67 AND L-07). With (A1) discharged, the marginally-bound condition (dr/dtau)ˆ2 = 1 - f needs no further input, and is verified by substitution at D=4..8 to integrate to rˆ(D-1) = 2 M alphaˆ2 sinhˆ2((D-1) tau/2 alpha) – P8 prop:cosmo's own law at D=4. Its deck group: sinhˆ2 has period i pi, so tau -¿ tau + 2 pi i alpha/(D-1) leaves rˆ(D-1) invariant and sends r -¿ eˆ{2 pi i/(D-1)} r (invariance verified at each D), so THE TURNAROUND FOLD IS Z_{D-1} – Z_3 at four, Z_4 at five. AND WHY BOTH THREES ARE D-1: clearing denominators, f = 0 is a polynomial of degree D-1 in r at every D, and the two threes are the two things one does with that ONE polynomial – the HORIZON three is its ZERO SET, the TURNAROUND three the DECK ACTION of the integral of 1-f. Distinct objects, one cause, and lem:twoturnings' non-identification is not in tension with it. HENCE the dimension result's fold step SURVIVES the split unchanged and structurally: the fold is a property of f, not of which structure reads it, so moving generation from one D-1 object to another cannot change a D-1 count. D=5 still gives four generations, now off Z_4 [borrowed from P3 / P8]). *Computes:* the law verified by substitution into the E=1 condition at D=4..8; the period's invariance checked symbolically at each D; the numerator degree computed *Bound:* THE SPLIT IS SHOWN ADMISSIBLE, NOT EXECUTED. Two costs: (i) the horizon three carries S_3 (Weyl/monodromy) and the turnaround three only Z_3 (cyclic deck), so on the split P14's family-symmetry sentence becomes a statement about COLOUR and flavour carries Z_3 – content, not damage, but a substantive revision (L-91); (ii) WHETHER R = gammaˆ5, the OFFSET parity, reaches the turnaround three AT ALL is not shown, and P14 needs the generations chiral – this is the split's falsifier and is UNRUN (L-92). The papers are NOT revised, and should not be until L-92 is run *Run:* `python3 receipts/P03_SdS_slicing/P03_the_weld_and_the_two_folds.py`

P14R22 `P14_the_isospin_exchange` [OK]
sec:correspondence / sec:charge (WHAT EXCHANGES uud AND udd, AND WHERE IT SITS. CLEAN NEGATIVE FIRST: charge conjugation ALONE cannot do it – w -¿ -w carries the quark class to the ANTIQUARK class (uud -¿ antibaryon), and no lap shift works either, because C reflects through the ORIGIN and the (u,d) pair is not centred there. THE UNIQUE INVOLUTION IS AFFINE: w -¿ 1/3 - w, SOLVED FOR rather than posited (c = u + d). AND IT IS THE STANDARD MODEL'S OWN MAP: with Q = I_3 + Y/2 the isospin Weyl reflection at fixed Y is Q -¿ Y - Q, and the constant came out equal to the quark doublet's hypercharge 1/3 with nothing fitted. ITS TWO FACTORS ARE BOTH THE CORPUS'S: w -¿ -w is P3's independent charge Z_2, w -¿ w+1/3 is ONE graze-point crossing = one wall. AND NEITHER IS IN D_6 – Aut(A_2) acts LINEARLY, so a map with nonzero offset cannot lie in it, which is exactly why P3 sec:charge's remark that the charge Z_2 is adjoined 'rather than sitting within it' is the structural precondition for the exchange to exist. DARYL'S 'THEY'RE EVEN THE OPPOSITE DIRECTION' IS THE PROOF: a reflection reverses direction and a translation does not, and u,d sit at +1/2 and -1/2 about the fixed point 1/6 [borrowed from P14 / P3]). *Computes:* the candidate maps tabulated and rejected; the involution solved for symbolically; the SM identity checked; the wall-count counterfactual run for N = 1..6 with the integrality condition solved *Bound:* THE FORCING IS TO A PAIR NOT A POINT: 3/(2N)+1/2 is integral at N = 1 AND N = 3, and N = 1 is excluded by CR's own one-hinge-truncation argument rather than by this computation. AND A REAL TENSION IS UNCOVERED: P3 sec:charge shows Q != 0 DESTROYS the r=0 branch point the whole matter sector stands on, so CR's winding charge and Maxwell's Q are NOT the same object and the involution inherits only the SHAPE of P3's Z_2. That debt is registered, not discharged *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_isospin_exchange.py`

P3R39 `P03_transverse_space_is_round` [OK]
sec:slicing / sec:chirality (THE TRANSVERSE SPACE IS THE ROUND Sˆ2, FORCED TWICE, AND THE TOWER DOES NOT TRUNCATE. CR's operator run with the transverse curvature k LEFT FREE gives the entire solution space f = k - 2M/r - Lambda rˆ2/3, so the operator alone does not fix k – exactly as L-90 said. BUT k is an additive constant independent of M, and CR's family passes through 2M=0 six times per swing where the cut IS the substrate (f = 1 - rˆ2/alphaˆ2, constant term

1), so k = +1 is an OUTPUT. AND the global form is fixed independently by SPIN STRUCTURE: no non-trivial subgroup of SO(3) acts freely on S^2 (every rotation has an axis), the only free finite action is the antipodal map, and its quotient RP^2 is non-orientable hence carries no spin structure – so Gamma = 1. AND the tower survives every option: even on an orbifold S^2/Z_n, lambda still runs over all positive integers and only the degeneracy thins from 2 lambda to ~2 lambda/n, because a quotient cannot change the local geometry that sets lambda. ** RECORDED AS A LOSS: L-15's bet that the seat count 12+3=15 explains one SM generation rested entirely on roundness being a choice, and L-15's own falfier (5) therefore fires ** [borrowed from P3 / P14]). *Computes:* G^t_t computed from the metric with a free-curvature transverse 2-space; dsolve for the entire kernel; the de Sitter member verified to solve it at Lambda = 3/alpha^2; the finite subgroups of SO(3) enumerated against free action; the Z_n-invariant degeneracies tabulated for n = 2,3,5 up to lambda = 8 *Bound:* WHAT IT DOES NOT DO: it does not explain the 12+3 = 15 agreement, and it closes the route that would have. It leaves L-15's TRIALITY grading (a lepton is a wall mode with lambda = 0 mod 3) entirely untouched, and it leaves L-88 – the tower's Kaluza-Klein shape – as the count's only surviving route *Run:* `python3 receipts/P03_SdS_slicing/P03_transverse_space_is_round.py`

P3R49 `P03_hexagon_null_triple` [OK]

sec:tour (THE HINGE-ENDS' CAUSAL TRICHOTOMY, AND WHY THE 2+1 IT CARRIES HAS NO ABSOLUTE ASSIGNMENT (Daryl's geometry; OBJECT CORRECTED by him at c54.19 – the c54.18 version of this receipt read the six as six independent hinges and was wrong). THE OBJECT: the hinge is the LINE the door swings on, at transverse 2 alpha, direction TIMELIKE, hence not one of the substrate's null rulings and not contained in it – it PIERCES the substrate once per horn at X0 = +-sqrt3 alpha. THREE hinges, TWO ends apiece; the six are 3x2. (And planes through a fixed LINE form a one-parameter pencil, which is why the construction has exactly one dial.) THE TRICHOTOMY: X.Y = alpha^2(-3 eps eps' + 4 cos(th_a - th_b)) classifies all fifteen pairs without remainder – TIMELIKE iff same hinge (3 pairs, X.Y = 7), spacelike iff same horn (6, X.Y = -5), NULL iff neither (6, X.Y = alpha^2). So every end is null-joined to exactly two, both opposite horn and both on OTHER hinges; two ends of one hinge CANNOT be null-separated since the hinge is timelike, so the pair a null binding appears to exclude IS THE HINGE ITSELF. The six chords project TWO-TO-ONE onto the three triangle sides (U_i-¿D_j and U_j-¿D_i, the two rulings told apart by horn), each tangent to the throat at its midpoint. THE 2+1: from any end the causal reach splits the three HINGES as {mine, timelike} against {the other two, null}. AND ITS ASSIGNMENT IS RELATIVE, PROVED: the symmetry group D_3 x Z_2 (order 12) acts TRANSITIVELY on the six ends and with ONE ORBIT on each causal class, so causal character is a COMPLETE invariant of a pair and nothing singles any end out [borrowed from P3]). *Computes:* all fifteen pairs, the orbit counts and the tangency verified exactly in sympy *Bound:* ASSERTS NOTHING ABOUT uud OR udd – and the c54.18 row's 'honest gap' (that it fails to show the two like members are alike as u and u are) is RETRACTED as a misreading: per Daryl the structure is just as well-posed as udd as it is uud, they are complementary, and species is relative in the same way, so a geometry that singled one out absolutely would be the failure. Where such a distinction would have to come from is quoted not derived: P3 sec:charge's charge Z_2 is INDEPENDENT of Aut(A_2)=D_6 and fixes every root. Registered L-59, untested *Run:* `python3 receipts/P03_SdS_slicing/P03_hexagon_null_triple.py`

`W1_the_two_plus_one_plus_one_is_excluded_by_a_symmetry_the_chiral_member_does_not_have` [OK]

sec:twofactors · sec:correspondence · sec:unpolarized (**THE MISSING FIFTH MULTIPLET IS EXCLUDED BY A SYMMETRY THE ACHIRAL MEMBER HAS, AND THE CHIRAL MEMBER PROVABLY DOES NOT.** P14 records as well-posed and declines the question whether T acts on one $\{}gamma^5$-eigenspace of the wall mode and trivially on the other, while its abstract states both the expected answer and a CAUSE — that the geometry built is the achiral member. **The causal claim is asserted in the paper and proved nowhere; it is proved here as a no-go.** If the handedness-exchanging involution $\{}sigma$ is a realised symmetry and commutes with T, then $T|_-=\{}sigma T|_+\{}sigma^{-1}$, so the two restrictions are conjugate and cannot differ in orbit structure — and $SU(2)_L$'s $2{+}1{+}1$ is exactly such a difference. **By exhaustion over the four admissible actions, both $2{+}1{+}1$ actions are excluded and the survivors are $2{+}2$ and $1{+}1{+}1{+}1$ — the shape P14 reports and defends.** The argument never uses colourlessness, so it forces the sector VECTOR-LIKE; and then the anomaly conditions go vacuous, returning the Standard Model hypercharges at $q=1/6$ on the chiral structure and nothing at all on the vector-like one. [borrowed from P14 / P11]). *Computes:* COMPUTES: that T commutes with the grading, and that ANY map of the ruling datum commutes with T (so $[\{}sigma,T]=0$ is P14's own independence, not an added hypothesis); the exhaustive four-action table with its orbit partitions; that exactly the $2{+}1{+}1$ actions admit no commuting handedness exchange and the excluded set is no larger; the explicit failure of the intertwining for each; that $SU(2)_L$ decomposes the four colourless states as $2{+}1{+}1$; the turning/non-turning polarisation criterion deciding when $\{}sigma$ exists; P11's built section carrying the disconnected component and the conserved twist; the anomaly conditions solved on both structures with the Standard Model hypercharges recovered on one and identical vanishing on the other; and the vector-like objection answered against itself. **Seeded both ways:** a defect planted in P11's conserved-charge statement makes it exit 1 naming that check, the restore verified by sha256 (42 checks) *Bound:* PAPER CLAIM AFFECTED — P14 sec:twofactors, where the question is recorded as unanswered. **NOT claimed:** that the fifth multiplet is delivered — lifting an obstruction is not producing a multiplet, and T's action on a chiral-member wall mode is NOT computed; that P14's hypercharge result is wrong — it states its own conditional plainly, and what is shown is what the conditional costs; that the enumeration covers linear representations carrying multiplicity — it is a set-level statement, which is P14's own framing. *Run:* `python3 receipts/L246_the_achirality_is_the_obstruction/W1_the_two_plus_one_plus_one_is_excluded_by_a_symmetry_`

P14R53 `P14_the_twist_conjugates_T_it_does_not_project_it` [OK]
sec:twofactors · sec:whichthree · sec:unpolarized (**PO-21, THE POSITIVE HALF, SETTLED IN THE NEGATIVE: the unpolarised (c != 0) cut PERMITS the 2+1+1 but does not SELECT it.** 'L-246' broke sigma with the twist c, making the 2+1+1 DEFINABLE; this shows the geometry does not FORCE it. ** T acts on the wall mode by pullback (invertible), so on a fixed mode content it CONJUGATES its c=0 action; "T trivial on a chirality block" is a conjugation invariant, so the 2+2 P14 reports is preserved – no turning, boost or shear moves it to 2+1+1. ** The mode content IS c-independent: the binding is radial (leaf measure dl = dr/sqrt(abs f), threshold s ¿ -3/4 from the near-wall f -¿ -2M/r) while the twist is the transverse omega, and the transverse space is the round Sˆ2 (P14 P03_transverse_space_is_round) on which the turning is a similarity, so lambda = j+1/2 does not split by chirality. ** THE TWIST CAN ROTATE T BUT NOT PROJECT IT ** – the lifting invariant (c, transverse, orbit-preserving) is orthogonal to the operation that would realise the split, a chiral projection trivialising T on one eigenspace, which neither the horn swap nor the twist is. Confirms and upgrades P14's standing "fails on the right-handed side" from a found gap to a theorem about the geometry, and closes the positive half P14 left well-posed. [borrowed from P14 / P11]). *Computes:* conjugation-invariance of triviality-on-a-block asserted for turnings and for 200 random invertible GL(2); the chirality-resolved action asserted 2+2 for every turning angle; the swap's eigenvalues (-1,+1) asserted preserved under conjugation; the radial binding threshold s ¿ -3/4 asserted set by the near-wall f and independent of the transverse twist; and gamma_z, gamma_t derived from the unpolarised cut's constraints and asserted EVEN under Q -¿ -Q *Bound:* NOT claimed – that the fifth multiplet is delivered (the geometry cannot; a chiral projection external to it is named as what a successor needs). THE ONE ESCAPE IS COMPUTED AND CLOSED: the twist's back-reaction on the Dirac measure is parity-even – the conformal factor gamma (fixed by quadrature from (P,Q)) depends on the twist only through omega_tˆ2, omega_zˆ2, so it is EVEN under Q -¿ -Q and the binding is identical on the +c and -c members, unable to depend on the sign of c (the chirality). No reversal premise stands *Run:* `python3 receipts/P14_matter_sector_paper/P14_the_twist_conjugates_T_it_does_not_project_it.py`

`C1_the_weyl_closure_is_generic_to_cubics_and_the_match_is_D3_alone` [OK]
corpus integrity (**THE CARTAN / HOLONOMY BAKE — first of the four fields 'L-272''s re-survey left outstanding, thrown r3164.** 'P07' and 'P14' report that the residue pairing's holonomy about the Nariai points, "adjoined to the walls' S_3, closes the Weyl group of $\{}so(6,\{}mathbb{C})$ — so the excess is substrate-derived rather than assumed." **CONFIRMED FROM SCRATCH:** continuing three roots of $rˆ3-r+2M$ and three branches of $\{}sqrt{f'(r_i)}$ around each Nariai point and around infinity gives transpositions, a 3-cycle, the thrice-punctured sphere's relation on the roots, and a group of **order 24** whose root image is S_3 and whose kernel is exactly 'P05''s Klein four-group

— element-order profile S_4's and no other order-24 group's, with no group theory put in. **BIT:** four further cubic families return the identical group, so **the closure is generic to one-parameter cubics and the horizon cubic is not special here.** "Substrate-derived rather than assumed" is sound about the derivation and misplaces the surprise: the group is forced by the cubic; what is not forced is that the substrate's Weyl group is the same group. **AND THAT MATCH IS D_3 ALONE** — among every classical and exceptional Weyl group of rank two or more, 24 selects $A_3=D_3$ and nothing else, the two coinciding by $\{}so(6,\{}mathbb{C})\{}cong\{}mathfrak{sl}(4,\{}mathbb{C})$. **A rank-three fact, in the shape of 'P03''s 'rem:dimension'.** **BOUNCED:** the two-transposition generation of the deck S_3 is 'P05''s 'prop:monodromy', with 'rem:monodromy-group' stating the condition a naive version misses. [borrowed from —]). *Computes:* COMPUTES: the root and square-root monodromy continued numerically around both Nariai points and around infinity, giving two transpositions, a 3-cycle, and $\{}gamma_+\{}gamma_-\{}gamma_\{}infty=1$ on the roots; the group generated, its order, its root image, its kernel and its element-order profile, identified as S_4; **controls in both directions** — a loop enclosing no Nariai point returns the identity and one lasso alone returns a group of order four, so 24 is a measurement — and robustness across base points and lasso radii; the same analysis on four further cubic families, all returning the identical group; the Weyl orders of A, B/C and D at ranks two through six against 24, and the exceptional orders; 'P05''s own propositions located verbatim; and the bundle apparatus surveyed word-bounded at ×0 (20 checks). *Bound:* NOT-A-PAPER-CLAIM — corpus integrity; no paper edited. **NOT claimed:** that the corpus is wrong — the group, the Klein four-group, the S_3 and the closure are all independently confirmed; that "substrate-derived" is false — the derivation is genuinely from the residue pairing, and genericity moves where the weight belongs rather than voiding it; that the match is a coincidence in the dismissive sense — it is an exact low-rank isomorphism, and the claim is that it is a fact about $\{}so(6,\{}mathbb{C})$ rather than about the cubic; that the missing apparatus is an error — what is owed is the name of a theorem; that the bake reached the CONNECTION — it computed the monodromy of the cubic, and the identification is the papers' own, stated as unreached in the ledger's boundary. *Run:* `python3 receipts/L273_the_cartan_bake/C1_the_weyl_closure_is_generic_to_cubics_and_the_match_is_D3_alone.py`

Appendix L The Knowledge Ledgers

Each claim marked L in the text rests on a field bake or the figure–theorem bake recorded in the named ledger, which ships with this work and carries the probes, the receipts, and the three registers kept apart — what bit, what bounced, and where the boundary is. A ledger is working apparatus, not a publication, and is cited here rather than in the bibliography. This appendix is generated from the ledger index, not maintained by hand.

L3 `CARTAN_HOLONOMY_LEDGER.md` [field bake]
The Cartan / connections-and-holonomy field-bake ledger — what bit, what bounced, and the boundary. First of the four fields 'L-272''s re-survey left outstanding. 'OWED' 622

L6 `COMBINATORICS_LEDGER.md` [field bake]
The combinatorics field-bake ledger — what bit, what did not, and why. Lane 8

L8 `CONFORMAL_GEOMETRY_LEDGER.md` [field bake]
conformal / Möbius geometry against the substrate — the field that refused, and why

L10 `FUNCTIONAL_ANALYSIS_LEDGER.md` [field bake]
The functional-analysis / unitarity field-bake ledger — the field that bounced, and the one routing fact it returned. Third of the four fields 'L-272''s re-survey left outstanding. 'OWED' 622

L18 `REPRESENTATION_THEORY_LEDGER.md` [field bake]
The representation-theory field bake — what bit, what bounced, and the boundary. The largest unbaked vocabulary in the corpus (×241 tight), thrown after the Phase 4 survey named it the standing first pick on measured usage

L19 `SPECTRAL_THEORY_LEDGER.md` [field bake]
The spectral-theory field bake — what bit, what bounced, and the boundary. Tier B's largest never-thrown field (×189 on the reach measure), verified as NOT covered by the harmonic ledger, which mentions spectral, self-adjoint and deficiency zero times

L13 `INDEX_THEORY_LEDGER.md` [field bake]
The differential-topology and index-theory field-bake ledger — what bit, what bounced, and where the boundary is

L14 `INTEGRABLE_SYSTEMS_LEDGER.md` [field bake]
The integrable-systems field-bake ledger — what bit, what bounced, and where the boundary is